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BUSINESS STATISTICS-II

B.Com II-Year IV-Sem.

As Per the Latest (2019-20) Syllabus of B.Com (OU) (CBCS)

B.Com : II-Year IV-Sem.

Exam QP's : **Latest QP :** Jan.-21 (OU)

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PREFACE

ABOUT THE SUBJECT

The subject **Business Statistics-II** includes all those topics which help to analyze the numerical information and figures on scientific basis. Statistics is defined in plural sense which means set of numerical data or figure. However, it carries out various functions and plays an important role in several areas like, economies, national income accounting, planning, business etc. Thus, statistics is used to gather, represent and examine the numerical figures on scientific basis.

The main objective behind introducing the subject '**Business Statistics-II**' in B.Com course is to make students acquire both theoretical and practical knowledge of the statistical topics.

The important topics discussed in this subject are as follows,

❖ Concept of Regression.

❖ Concept of Index Numbers.

❖ Concept of Time Series.

❖ Concept of Probability with Approaches and Theorems.

❖ Concept of Binomial, Poisson and Normal Distribution.

ABOUT THE BOOK

The book entitled **'Business Statistics-II'** is designed for B.Com II-Year IV-Semester students. The content provided in this book is strictly as per the latest syllabus prescribed by Osmania University.

Every concept is explained in a simple manner with sufficient number of examples so as to facilitate better understanding and easy learning in a shorter span of time. Keeping in view the examination pattern of B.Com students, this book provides the following features,

❖ Every unit is structured into two main sections viz., **Short Questions (Part-A)** and **Essay Questions (Part-B)** with Answers.

❖ **List of Important Definitions and Formulae** are given.

❖ **Previous to Latest Exam Question Papers with Answers** are included in the units matching and attached at the end of this book.

❖ **Unit-wise Frequently Asked Questions** and **Important Questions** are included to help students prepare for Internal and External Assessment.

❖ **Three Model Papers** are provided in order to help students understand the paper pattern in the end examination.

An attempt has been made through this book to present theoretical and practical knowledge of "**Business Statistics-II**". This book is especially prepared for undergraduate students.

The table below illustrates the complete idea about the subject, which will be helpful to plan and score good marks in the end examinations.

S.No.	Unit Name	Description
1.	Regression	This unit covers the topics: Introduction - Linear and Non Linear Regression – Correlation Vs. Regression - Lines of Regression - Derivation of Line of Regression of Y on X - Line of Regression of X on Y - Using Regression Lines for Prediction.
2.	Index Numbers	This unit covers the topics: Introduction - Uses - Types - Problems in the Construction of Index Numbers - Methods of Constructing Index Numbers - Simple and Weighted Index Number (Laspeyre - Paasche, Marshall – Edgeworth) - Tests of Consistency of Index Number: Unit Test - Time Reversal Test - Factor Reversal Test - Circular Test - Base Shifting - Splicing and Deflating of Index Numbers.
3.	Time Series	This unit covers the topics: Introduction - Components – Methods-Semi Averages - Moving Averages – Least Square Method - Deseasonalisation of Data – Uses and Limitations of Time Series.
4.	Probability	This unit covers the topics: Probability – Meaning - Experiment – Event - Mutually Exclusive Events - Collectively Exhaustive Events - Independent Events - Simple and Compound Events - Basics of Set Theory – Permutation – Combination - Approaches to Probability: Classical – Empirical – Subjective - Axiomatic - Theorems of Probability: Addition – Multiplication - Baye's Theorem.
5.	Theoretical Distributions	This unit covers the topics: Binomial Distribution: Importance – Conditions – Constants - Fitting of Binomial Distribution. Poisson Distribution: – Importance – Conditions – Constants - Fitting of Poisson Distribution. Normal Distribution: – Importance - Central Limit Theorem - Characteristics – Fitting a Normal Distribution (Areas Method Only).

It is sincerely hoped that this book will satisfy the expectations of students and at the same time helps them to score maximum marks in exams.

Suggestions for improvement of the book from our esteemed readers will be highly appreciated and incorporated in our forthcoming editions.

BUSINESS STATISTICS-II

B.Com II-Year IV-Semester (OU)

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LIST OF IMPORTANT DEFINITIONS AND FORMULAE

UNIT - I

1. According to M.M. Blair, "Regression is a mathematical measure of the average relationship between two or more variables in terms of the original units of the data".
2. According to Ya-Lun Chou, "Regression analysis attempts to establish the 'nature of the relationship' between variables that is, to study the functional relationship between the variables and thereby provide a mechanism for prediction or forecasting".
3. Linear regression is a form of regression which is used for modeling the relationship between scalar variables like 'X' and 'Y'.
4. In the non-linear regression the explained variable (dependent variable) changes at varying rate with a given change in the explaining variable (independent variable).
5. Correlation is a measure of the 'degree and direction' of relationship between the variables.
6. In a bi-variate distribution, if the variables are related then the points when plotted in the scatter diagram will lie near a straight line which is called the line of regression and the regression is said to be linear regression.
7. The regression coefficient X on Y measures the change in X corresponding to a unit change in Y and the regression coefficient of Y on X measures the change in Y corresponding to a unit change in X,

   $r.\frac{\sigma_y}{\sigma_x}$ *and* $r.\frac{\sigma_x}{\sigma_y}$ are known as coefficient of regression.

8. "$r.\frac{\sigma_y}{\sigma_x}$" is denoted by "$b_{xy}$" and is given by,

   $$b_{xy} = \frac{\sum(x-\bar{x})(y-\bar{y})}{\sum(x-\bar{x})^2}$$

9. The predicted value of variable 'Y' (Y_i) for a given variable 'X' (X_i) will be represented as,

   $Y_i = a + bX_i$

10. Regression equation of X on Y

    $$x - \bar{x} = r\left[\frac{\sigma_x}{\sigma_y}\right](y - \bar{y})$$

11. Regression equation of Y on X

    $$y - \bar{y} = r\left[\frac{\sigma_y}{\sigma_x}\right](x - \bar{x})$$

UNIT - II

1. According to Maslow, "Index number is a numerical value characterizing the change in complex economic phenomena over a period of time or space".
2. Single Price Index = $\frac{\text{Current value}}{\text{Base value}} \times 100$ or $\frac{P_1}{P_0} \times 100$
3. In a weighted aggregate price index, certain weight is assigned to each and every commodity or item of group in accordance with its significance.
4. Laspeyre's index method was introduced by a famous statistician named "Laspeyre's". In this method, prices of all items or commodities are weighted by the quantity, consumed both in the base and the current year. The formula for Laspeyre's price index method is as follows,

   $$P_{01} = \frac{\Sigma p_1 q_0}{\Sigma p_0 q_0} \times 100$$

5. In Paasche's index method, prices are weighted by the quantity consumed in the current year, rather than considering quantity consumed in base year. The formula for Paasche's index method is,

$$P_{01} = \frac{\Sigma p_1 q_1}{\Sigma p_0 q_1} \times 100$$

6. In Marshall-Edgeworth's index method, both the current year as well as base year quantities are considered as to calculate the index. The formula for Marshall-Edgeworth's Index method is as follows,

$$I_p\text{ (ME)} = \frac{\Sigma(q_0 + q_1)P_1}{\Sigma(q_0 + q_1)P_0} \times 100$$

$$= \left(\frac{\Sigma P_1 q_0 + \Sigma P_1 q_1}{\Sigma P_0 q_0 + \Sigma P_0 q_1}\right) \times 100$$

7. A statistician named "Fisher" introduced this method, which is a geometric mean of Laspeyre's and Paasche's methods. The formula used for this method is,

$$P_{01} + \sqrt{\frac{\Sigma p_1 q_0}{\Sigma p_0 q_0} \times \frac{\Sigma p_1 q_1}{\Sigma p_0 q_1}} \times 100$$

8. According to unit test, the formula of index number should be independent of the units under which prices and quantities are quoted.

9. Time reversal test is basically used for checking whether the selected method would work for both forward and backward. According to this test, the formula should give exact ratio when we compare one point with the another i.e., for example,

$$P_{01} = \frac{1}{P_{01}} \text{ or } P_{01} \times P_{10} = 1$$

$$Q_{01} \times Q_{10} = 1$$

10. According to circular test, if P_{ab} is the price index whose base year is 'a' and 'b' is the period, P_{bc} is the price index with base year 'b' and 'c' is the period and P_{ca} is the price index whose base year is 'c' and 'a' is the period then,

$$P_{ab} \times P_{bc} \times P_{ca} = 1$$

11. Base shifting refers to the process of shifting a base period of an index.

12. Splicing is the process of combining two or more index numbers which may convert various bases into a single series.

13. Deflating refers to the process of making allowances for the impact of changing prices. A rise in price level means a reduction in the purchasing power of money.

UNIT - III

1. Ya-Lun-Chou has defined "time series as a collection of readings belonging to different time periods, of some economic variable or composite of variables".

2. In moving average method, the average value for a number of years (month or weeks) is secured and this average is taken as the normal or trend value for the unit of time falling at the middle of the period covered in the calculation of the average.

3. Formula for 3 yearly moving average will be,

$$\frac{a+b+c}{3}, \frac{b+c+d}{3}, \frac{c+d+e}{3} \ldots$$

4. Formula for 5 yearly moving average will be,

$$\frac{a+b+c+d+e}{5}, \frac{b+c+d+e+f}{5}, \frac{c+d+e+f+g}{5} \ldots$$

5. Least square method is a statistical procedure which is used to find the best fit curve for the set of data where different variables are involved.

6. Deseasonalisation of data refers to the process of eliminating seasonal fluctuations from the given time series for, it facilitates in adjusting the given time series for seasonal fluctuations and therefore left out with variables like trend component, cyclical and irregular variations.

UNIT - IV

1. Probability can be defined as the chance or 'likelihood of occurrence' of an experiment or event. Probability of any event ranges from 0 to 1.

2. A joint probability is the probability of occurrence of two or more simple events. It is the product of two marginal probabilities.

3. Joint probability of A and B is represented as $P(A \cap B)$

   $\therefore \quad P(A \cap B) = P(A).P(B)$

4. The number of ways of selecting some objects (r) from total number of distinguishable objects (D) which can be arranged in an order is called 'permutation'. It is denoted as '${}^nP_r$'.

$$\boxed{{}^nP_r = \frac{n!}{(n-r!)}}$$

5. The number of ways of selecting 'r' objects from 'n' different objects irrespective of their arrangement is called as 'combinations'. It is denoted as ${}^nC_r$.

$$\boxed{{}^nC_r = \frac{n!}{r!(n-r)!}}$$

6. An event is a possible outcome of an experiment or a result of trial. Basically there are two types of events. Simple and compound event.

7. The collection of finite or infinite number of objects with some common property is called set. The objects belonging to the set are called members or elements of the set.

8. According to priori approach in a random experiment when there are 'm' favourable cases when a favourable event A occurs and 'n' total possible cases when favourable event A does not occur. Then the probability of getting favourable cases $P(A)$ can be calculated as,

$$P(A) = \frac{\text{Number of favourable cases}}{\text{Total number of possible outcomes}} = \frac{m}{n}$$

9. Baye's theorem got its name from the British mathematician, 'Thomas Bayes' in 1763. Baye's theorem deals with revising of priori probability by making use of new information for calculating posterior probabilities.

10. Posterior probabilities can be calculated by using the following formula,

$$\boxed{P(A_i / B) = \frac{P(A_i \cap B)}{\sum_{i=1}^{n} (A_i \cap B)} = \frac{P(B / A_i)P(A_i)}{\sum_{i=1}^{n} P(B/A_i)\ P(A_i)}}$$

(or)

$$\boxed{P(A_i / B) = \frac{P(A_i \cap B)}{P(B)}}$$

UNIT - V

1. Binomial distribution is a discrete probability distribution developed by a Swiss mathematician, 'James Bernoulli' in 1700. It is used for finite or limited number of trails 'n'. It produces successes and failures based on two parameters 'n' and 'p'.

2. The mean of binomial distribution,

$$\mu = np$$

3. Standard deviation is given by,

$$\sigma = \sqrt{npq}$$

4. The random variable 'x' the number of successes 'r' in n trials has a probability distribution.

   $P(x = r) = {}^nC_r\, p^r\, q^{n-r}$

5. Poisson distribution can be used generally to approximate the binomial distribution when 'n' value is large (i.e., $n \to \infty$) and 'p' value is very small (i.e., $p \to o$)

6. Always the sum of infinite probabilites in poisson distribution is 1 i.e.,

$$P(0) + P(1) + P(2) + ... + P(\infty) = 1$$

7. The probability of 'X' occurrences in poisson distribution is given by,

$$P(X) = \frac{e^{-\lambda}.\lambda^x}{x!}$$

8. In normal distribution, the probability of occurrence of values of random variables are calculated within a range or interval whereas the probability of a particular value cannot be calculated and is always assumed to be zero. The random variable 'X' is given as follows,

$$F(X) = \frac{1}{\sigma\sqrt{2\pi}} e^{\frac{-1}{2}\left(\frac{X-\mu}{\sigma}\right)^2}$$

9. Central limit theorem states that the distribution of the sum of i.i.d (independently and identically distributed) random variables will be normal asymptotically under general conditions with mean $\mu = \sum_{i=1}^{n} \mu_i$ and standard deviation σ where $\left(\sigma^2 = \sum_{i=1}^{n} \sigma_1^2\right)$.

UNIT 1 Regression

SHORT QUESTIONS

Q1. Define Regression Analysis.

Answer : Important Question

For answer refer Unit-I, Page No. 2, Q.No. 1.

Q2. Features of Regression Coefficients.

Answer : May/June-19, Q1(MGU)

For answer refer Unit-I, Page No. 2, Q.No. 2.

Q3. Write three limitations of regression analysis.

Answer : May/June-18, Q1(a) (KU)

For answer refer Unit-I, Page No. 3, Q.No. 5.

Q4. If r = 0.8; σ_x = 2.5, σ_y= 3.5, find b_{xy} and b_{yx}.

Answer : May/June-19, Q1 (OU)

For answer refer Unit-I, Page No. 4, Q.No. 6.

Q5. Co-efficient of correlation = 0.60, σ_x = 1.5, σ_y = 2.0, x = 10, y = 20, find regression equation y on x.

Answer : May/June-18, Q1(g) (KU)

For answer refer Unit-I, Page No. 4, Q.No. 8.

ESSAY QUESTIONS

Q1. What is regression analysis? Explain regression variables and types of regression.

Answer : Important Question

For answer refer Unit-I, Page No. 5, Q.No. 9.

Q2. Write the relation between correlation and regression.

Answer : May/June-18, Q2(a) (KU)

For answer refer Unit-I, Page No. 6, Q.No. 11.

Q3. Define regression and what are the differences between correlation and regression.

Answer : May/June-18, Q9(a) (OU)

For answer refer Unit-I, Page No. 7, Q.No. 13.

Q4. From the following data obtain the two regression equations and calculate the Correlation Co-efficient.

X:	2	4	6	8	10	12	14	16	18
Y:	18	16	20	24	22	26	28	32	30

Answer : May/June-19, Q9(a) (OU)

For answer refer Unit-I, Page No. 11, Q.No. 17.

Q5. **Following are the marks in Statistics and English in an Annual Examination.**

	Statistics (X)	English (Y)
Mean	40	50
Standard Derivation	10	16
Co-efficient Correlation	0.5	

(i) Estimate the score of English, when the score in Statistics is 50.

(ii) Estimate the score of Statistics, when the score in English is 30.

Answer : May/June-19, Q9(b) (OU)

For answer refer Unit-I, Page No. 12, Q.No. 18.

Q6. **Given:**

$\Sigma x = 56$, $\Sigma y = 40$, $\Sigma x^2 = 524$, $\Sigma y^2 = 256$, $\Sigma xy = 364$, $N = 8$

(i) Find the two Regression equations and

(ii) The Correlation Coefficient.

Answer : May/June-18, Q9(b) (OU)

For answer refer Unit-I, Page No. 15, Q.No. 21.

Q7. **You are given the following information about advertisement expenditure and sales:**

	Adv. Exp (X) (₹ Crores)	Sales (Y) (₹ Crores)
Mean	20	120
S.D	5	25
Correlation coefficient	0.8	

1. Calculate two regression equations.
2. Find likely sales when Adv. Expenses is ₹ 25 Crores.
3. What should be the Adv. Budget if the company wants to attain sales target of ₹ 150 crores

Answer : Oct./Nov.-13, Q12(b) (OU)

For answer refer Unit-I, Page No. 18, Q.No. 24.

UNIT 2 Index Numbers

SHORT QUESTIONS

Q1. **Define Index Numbers.**

Answer : Important Question

For answer refer Unit-II, Page No. 26, Q.No. 1.

Q2. **Importance of Index Numbers** May/June-18, Q2 (OU)

OR

Advantages of Index Numbers March/April-17, Q8 (OU)

OR

Explain the uses of Index Numbers.

Answer : March/April-14, Q7 (OU)

For answer refer Unit-II, Page No. 26, Q.No. 2.

Q3. Types of Index Numbers

Answer : (May/June-19, Q2 (OU) | Oct./Nov.-14, Q4 (OU))

For answer refer Unit-II, Page No. 26, Q.No. 3.

Q4. Calculate Index number by Average Price Relative Method by using Arithmetic Mean.

Commodity	P	Q	R	S	T
Price 2017	2	6	10	5	12
Price 2018	4	8	15	5	8

Answer : May/June-19, Q3 (OU)

For answer refer Unit-II, Page No. 28, Q.No. 9.

Q5. Compute price index by weighted average of price relatives method using arithmetic mean.

Commodities	A	B	C	D	E
Price in base year	10	6	14	22	18
Quantity in base year	160	180	120	40	80
Price in current year	16	8	14	28	24

Answer : March/April-15, Q8 (OU)

For answer refer Unit-II, Page No. 29, Q.No. 10.

ESSAY QUESTIONS

Q1. Define Index Number. What are its features and uses?

May/June-19, Q10(a) (OU)

OR

What is Index Number? Write importance of index numbers.

Answer : May/June-18, Q3(a) (KU)

For answer refer Unit-II, Page No. 30, Q.No. 11.

Q2. What is the importance and limitations of index numbers? Explain.

Answer : May/June-19, Q7(b) (MGU)

For answer refer Unit-II, Page No. 30, Q.No. 12.

Q3. From the following data, calculate price index number by,

(i) Laspeyre's method

(ii) Paasche's method

(iii) Fisher's ideal method

(iv) Kelly's method

(v) Walsch's method

(vi) Marshall Edgeworth's method

(vii) Dorbish and Bowleys method.

Commodity	2005		2009	
	Price ₹	Qty	Price ₹	Qty
A	20	8	40	6
B	50	10	60	5
C	40	15	50	15
D	20	20	20	25

Answer : Important Question

For answer refer Unit-II, Page No. 37, Q.No. 21.

Q4. From the following data, calculate price index number by,

(i) Laspeyre's

(ii) Paasche's

(iii) Fisher's ideal method.

Commodity	2005		2009	
	Price ₹	Qty	Price ₹	Qty
A	20	8	40	6
B	50	10	60	5
C	40	15	50	15
D	20	20	20	25

Answer : (March/April-12, Q13(b) (OU) | March/April-11, Q13(b) (OU))

For answer refer Unit-II, Page No. 39, Q.No. 23.

Q5. From the following data calculate Price Index Number by using (i) Paasche's Method and (ii) Marshal Edgeworth Method.

Item	Base Year		Current Year	
	Price (₹)	Expenditure (₹)	Price (₹)	Expenditure (₹)
P	6	300	10	560
Q	2	200	2	240
R	4	240	6	360
S	10	300	12	288
T	3	120	8	240

Answer : May/June-18, Q10(b) (OU)

For answer refer Unit-II, Page No. 41, Q.No. 25.

Q6. Calculate Fisher's Ideal Index Number and test whether it satisfies Time Reversal and Factor Reversal Test for the following data.

Commodity	Base year		Current year	
	Price (₹)	Qty (kg)	Price (₹)	Qty (kg)
A	32	50	30	50
B	30	35	25	40
C	16	55	18	50

Answer : Oct./Nov.-16, Q13(b) (OU)

For answer refer Unit-II, Page No. 45, Q.No. 28.

Q7. The following are the indices (2007, Base):

Year	2007	2008	2009	2010	2011	2012	2013	2014	2015	2016	2017
Indices	100	120	122	116	120	120	137	136	149	156	137

Shift the base to 2012 and recast the index numbers.

Answer : May/June-18, Q10(a) (OU)

For answer refer Unit-II, Page No. 48, Q.No. 31.

UNIT 3 Time Series

SHORT QUESTIONS

Q1. What is Time Series?

Answer : Important Question

For answer refer Unit-III, Page No. 64, Q.No. 1.

Q2. Utility of Time Series Analysis. May/June-19, Q4 (OU)

OR

What are the uses of time series? May/June-18, Q4 (OU)

Answer :

For answer refer Unit-III, Page No. 64, Q.No. 2.

Q3. Objectives of Time Series

Answer : May/June-19, Q3 (MGU)

For answer refer Unit-III, Page No. 64, Q.No. 3.

Q4. What is seasonal variations? Write three features of seasonal variations.

Answer : May/June-18, Q1(c) (KU)

For answer refer Unit-III, Page No. 65, Q.No. 4.

Q5. Fit a trend line to the following data by the Freehand Method.

Year	2009	2010	2011	2012	2013	2014	2015	2016	2017
Sales (₹)	19	22	24	20	23	25	23	26	25

Answer : May/June-19, Q5 (OU)

For answer refer Unit-III, Page No. 65, Q.No. 6.

Q6. Annual trend of Milk Consumption (Y) is 18.6 + 1.8X. Convert the equation into monthly basis.

Answer : March/April-17, Q7 (OU)

For answer refer Unit-III, Page No. 66, Q.No. 7.

ESSAY QUESTIONS

Q1. What is Time Series? Explain the components of time series.

Answer : Important Question

For answer refer Unit-III, Page No. 68, Q.No. 10.

Q2. Write the features of tendency.

Answer : May/June-18, Q4(a) (KU)

For answer refer Unit-III, Page No. 69, Q.No. 11.

Q3. From the following data, calculate trend values using Four Yearly Moving Averages,

Year	2009	2010	2011	2012	2013	2014	2015	2016	2017
Production (in tons)	506	620	1036	673	588	696	1116	738	663

Answer : (May/June-19, Q11(a) (OU) | Oct./Nov.-12, Q13(a) (OU))

For answer refer Unit-III, Page No. 73, Q.No. 15.

Q4. What is the least square method? What are the merits and demerits of this method?

Important Question

OR

What is the least square method and explain its advantages and disadvantages?

Answer : May/June-19, Q8(b) (MGU)

For answer refer Unit-III, Page No. 75, Q.No. 18.

Q5. Below are given the figures of production (in thousand quintals) of a sugar factory.

Years	2001	2002	2003	2004	2005	2006	2007
Production	77	88	94	85	91	98	90

(i) Fit a straight line by the least squares method and tabulate the trend values.

(ii) What is the yearly increase in the production of sugar?

March/April-14, Q13(b) (OU)

OR

Fit a straight line by the Least Square Method and tabulate the trend values for the above data.

Year	2011	2012	2013	2014	2015	2016	2017
Production (in tons)	77	88	94	85	91	98	90

Answer : May/June-19, Q11

For answer refer Unit-III, Page No. 76, Q.No. 19.

Q6. From the following data calculate trend values based on least square method and estimate production in 1993.

Year	1982	1983	1984	1985	1986	1987	1988	1989
Production (000 tonnes)	58	56	55	51	47	38	35	32

Answer : May/June-19, Q8(a) (MGU)

For answer refer Unit-III, Page No. 79, Q.No. 22.

Q7. State the uses and limitations of time series.

Answer : Important Question

For answer refer Unit-III, Page No. 84, Q.No. 26

UNIT 4 Probability

SHORT QUESTIONS

Q1. Define Probability.

Answer : Important Question

For answer refer Unit-IV, Page No. 92, Q.No. 1.

Q2. What are mutually exclusive events, non-mutually exclusive events and dependent events?

Important Question

OR

Explain (i) Mutually exclusive events and (ii) Not-mutually exclusive events.

May/June-18, Q5 (OU)

OR

Explain:

(i) Mutually exclusive events and

(ii) Dependent events.

Answer : May/June-19, Q6 (OU)

For answer refer Unit-IV, Page No. 92, Q.No. 2.

Q3. When two dice are thrown, find the probability that the sum of the numbers is either 10 or 11.

Answer : May/June-18, Q6 (OU)

For answer refer Unit-IV, Page No. 93, Q.No. 6.

Q4. n(A) = 35, n(B) = 30, n(A ∩ B) = 20 then find ∩ (A ∪ B).

Answer : May/June-18, Q1(d) (KU)

For answer refer Unit-IV, Page No. 94, Q.No. 8.

Q5. How many 3 letter words can be formed from the English world "SUCCESS"?

Answer : May/June-18, Q1(e) (KU)

For answer refer Unit-IV, Page No. 94, Q.No. 9.

Q6. Find the value of $6P_4$, $5P_2$.

Answer : May/June-18, Q1(f) (KU)

For answer refer Unit-IV, Page No. 94, Q.No. 10.

ESSAY QUESTIONS

Q1. What do you mean by probability? Explain the importance of probability.

Answer : Important Question

For answer refer Unit-IV, Page No. 96, Q.No. 12.

Q2. What are the key concepts of probability?

Answer : May/June-18, Q5(a) (KU)

For answer refer Unit-IV, Page No. 96, Q.No. 13.

Q3. How many ways are there to paste 2 photos on notice board from a group of 6 photos?

Answer : May/June-18, Q5(b) (KU)

For answer refer Unit-IV, Page No. 103, Q.No. 22.

Q4. Two dice are rolled. Find the probability of 6 number event when two dice are rolled.

Answer : May/June-19, Q9(b) (MGU)

For answer refer Unit-IV, Page No. 106, Q.No. 26.

Q5. Explain the probability theorem basic concpets

May/June-19, Q9(a) (MGU)

OR

Explain the major theorems of probability.

Answer : Important Question

For answer refer Unit-IV, Page No. 107, Q.No. 28.

Q6. From 30 tickets marked with first 30 numerals, 1 ticket i6s drawn at random. It is then replaced and a second draw is made. Find the probability that in the first draw it is multiple of 5 or 7 and in second draw it is a multiple of 3 or 7.

Answer : May/June-18, Q12(a) (OU)

For answer refer Unit-IV, Page No. 109, Q.No. 30.

Q7. A company has two Plants for manufacturing Scooters. Plant I manufactures 80% of the Scooters and Plant II manufactures 20%. At the Plant I 85% Scooters are rated to be of standard quality and at Plant II 65% Scooters are rated to be of standard quality. One Scooter was selected at random. What is the probability that

(i) It is manufactured by Plant I

(ii) It is manufactured by Plant II – which is of standard quality.

Answer : May/June-19, Q12(b) (OU)

For answer refer Unit-IV, Page No. 112, Q.No. 34.

Q8. In a certain college 25% of boys and 10% of girls are studying mathematics. The girls constitute 60% of the students. If a student is selected at random and is found to be studying mathematics, find the probability that the student is a (i) girl (ii) boy.

Answer : Important Question

For answer refer Unit-IV, Page No. 115, Q.No. 36.

UNIT 5 Theoretical Distributions

SHORT QUESTIONS

Q1. Bring out the differences between Binomial and Poisson distribution.

Answer : Important Question

For answer refer Unit-V, Page No. 124, Q.No. 2.

Q2. Comment on the following:

For a Binomial Distribution Mean = 7 and Variance = 11.

Answer : May/June-19, Q8 (OU)

For answer refer Unit-V, Page No. 124, Q.No. 4.

Q3. Properties of Normal Distribution.

Answer : May/June-18, Q8 (OU)

For answer refer Unit-V, Page No. 125, Q.No. 5.

Q4. 6 coins are tossed at the same time find the probability that 4 heads are occurred.

Answer : May/June-19, Q5 (MGU)

For answer refer Unit-V, Page No. 125, Q.No. 6.

Q5. What is the probability of getting 3 heads when a coin is tossed 5 times?

Answer : May/June-18, Q1(h) (KU)

For answer refer Unit-V, Page No. 126, Q.No. 8.

ESSAY QUESTIONS

Q1. What is Binomial distribution? State its importance, applications and assumptions.

Answer : Important Question

For answer refer Unit-V, Page No. 127, Q.No. 9.

Q2. Ten unbiased coins are tossed simultaneously. Find the probability of obtaining.

(i) Exactly 6 Heads

(ii) Atleast 8 Heads

(iii) No Heads

(iv) Atleast one Head

(v) Not more than 3 Heads and

(vi) Atleast 4 heads.

Answer : May/June-19, Q13(a) (OU)

For answer refer Unit-V, Page No. 132, Q.No. 14.

Q3. Five coins are tossed 3,200 times, find the frequencies of the distribution of heads and tails; and tabulate the results and also calculate Mean and Standard Deviation of fitted distribution.

Answer : May/June-18, Q13(a) (OU)

For answer refer Unit-V, Page No. 134, Q.No. 15.

Q4. Explain about the properties and applications of poisson distribution.

Important Question

OR

Explain the features of Poisson distribution.

Answer : May/June-19, Q10(a) (MGU)

For answer refer Unit-V, Page No. 136, Q.No. 18.

Q5. Fit a Poisson distribution to the following data:

X	0	1	2	3	4
Y	211	90	19	5	0

($e^{-m} = 0.6443$)

Answer : May/June-19, Q13(b) (OU)

For answer refer Unit-V, Page No. 138, Q.No. 21.

Q6. What is normal distribution? State its characteristics. Important Question

OR

What is normal distribution? Write its any five features.

Answer : May/June-18, Q6(a) (KU)

For answer refer Unit-V, Page No. 144, Q.No. 28.

Q7. A study of past participants indicates that the mean length of time spent on the programme is 500 hours; and that, this normal distribution random variable has a standard deviation of 100 hours. What is the probability that a participant selected at random will required to complete the programme in following cases:

(i) 'More' than 500 hrs

(ii) Between 500 and 650 hrs

(iii) Between 550 and 650 hrs

(iv) Less than 580 hrs

(v) Between 420 and 570 hrs.

Answer : May/June-18, Q13(b) (OU)

For answer refer Unit-V, Page No. 146, Q.No. 30.

MODEL PAPER 1

FACULTY OF COMMERCE

B.Com. IV-Semester (CBCS) Examination

BUSINESS STATISTICS-II

Time: 3 Hours | Max. Marks: 80

PART - A (5 × 4 = 20 Marks)

[Short Answer Type]

Note: Answer any FIVE of the following questions.

1. Define Regression Analysis. (Unit-I, Page No. 2, Q1)

2. Importance of Index Numbers (Unit-II, Page No. 26, Q2)

3. Calculate index number by simple aggregative method. (Unit-II, Page No. 27, Q6)

	A	B	C	D
Price in 2005 (₹)	162	256	257	132
Price in 2007 (₹)	171	164	189	145

4. Utility of Time Series Analysis. (Unit-III, Page No. 64, Q2)

5. What are mutually exclusive events, non-mutually exclusive events and dependent events? (Unit-IV, Page No. 92, Q2)

6. When two dice are thrown, find the probability that the sum of the numbers is either 10 or 11. (Unit-IV, Page No. 93, Q6)

7. Properties of Normal Distribution. (Unit-V, Page No. 125, Q5)

8. Comment on the following:

   For a Binomial Distribution Mean = 7 and Variance = 11. (Unit-V, Page No. 124, Q4)

PART - B (5 × 12 = 60 Marks)

(Essay Answer Type)

Note: Answer ALL the questions.

9. (a) Define regression and what are the differences between correlation and regression. (Unit-I, Page No. 7, Q13)

OR

(b) Following are the marks in Statistics and English in an Annual Examination.

	Statistics (X)	English (Y)
Mean	40	50
Standard Derivation	10	16
Co-efficient Correlation	0.5	

(i) Estimate the score of English, when the score in Statistics is 50.

(ii) Estimate the score of Statistics, when the score in English is 30. (Unit-I, Page No. 12, Q18)

10. (a) What is the importance and limitations of index numbers? Explain. (Unit-II, Page No. 30, Q12)

OR

(b) Compute Price Index Number by using:

(i) Paasches and

(ii) Marshal and Edgeworth methods. (Unit-II, Page No. 39, Q22)

Commodity	Base Year		Current Year	
	Price	Quantity	Price	Quantity
P	5	100	6	150
Q	4	80	5	100
R	2	60	5	72
S	12	30	9	33

11. (a) Write the features of tendency. (Unit-III, Page No. 69, Q11)

OR

(b) From the following data, calculate trend values using Four Yearly Moving Averages, (Unit-III, Page No. 73, Q15)

Year	2009	2010	2011	2012	2013	2014	2015	2016	2017
Production (in tons)	506	620	1036	673	588	696	1116	738	663

12. (a) What are the key concepts of probability? (Unit-IV, Page No. 96, Q13)

OR

(b) A company has two Plants for manufacturing Scooters. Plant I manufactures 80% of the Scooters and Plant II manufactures 20%. At the Plant I 85% Scooters are rated to be of standard quality and at Plant II 65% Scooters are rated to be of standard quality. One Scooter was selected at random. What is the probability that

(i) It is manufactured by Plant I

(ii) It is manufactured by Plant II – which is of standard quality.

(Unit-IV, Page No. 112, Q34)

13. (a) What is Binomial distribution? State its importance, applications and assumptions. (Unit-V, Page No. 127, Q9)

OR

(b) Fit a Poisson distribution to the following data:

X	0	1	2	3	4
Y	211	90	19	5	0

($e^{-m} = 0.6443$)

(Unit-V, Page No. 138, Q21)

MODEL PAPER 2

FACULTY OF COMMERCE

B.Com. IV-Semester (CBCS) Examination

BUSINESS STATISTICS-II

Time: 3 Hours Max. Marks: 80

PART - A (5 × 4 = 20 Marks)

[Short Answer Type]

Note: Answer any FIVE of the following questions.

1. If $\gamma = 0.6$, $\sigma_x = 1.5$ and $\sigma_y = 2$, find the b_{xy} and b_{yx}. (Unit-I, Page No. 4, Q7)

2. Define Index Numbers. (Unit-II, Page No. 26, Q1)

3. From the following data calculate a price index based on price relatives method using Arithmetic Mean. (Unit-II, Page No. 28, Q8)

Commodity	A	B	C	D	E	F
Price 2015 (₹)	45	60	20	50	85	120
Price 2016 (₹)	55	70	30	75	90	130

4. What is seasonal variations? Write three features of seasonal variations. (Unit-III, Page No. 65, Q4)

5. Fit a trend line to the following data by the Freehand Method. (Unit-III, Page No. 65, Q6)

Year	2009	2010	2011	2012	2013	2014	2015	2016	2017
Sales (₹)	19	22	24	20	23	25	23	26	25

6. $n(A) = 35$, $n(B) = 30$, $n(A \cap B) = 20$ then find $\cap (A \cup B)$. (Unit-IV, Page No. 94, Q8)

7. Calculate probability of 53 Mondays in a leap year. (Unit-IV, Page No. 95, Q11)

8. 6 coins are tossed at a time, what is the probability of obtaining 4 or more heads? (Unit-V, Page No. 125, Q7)

PART - B (5 × 12 = 60 Marks)

(Essay Answer Type)

Note: Answer ALL the questions.

9. (a) From the following data obtain the two regression equations and calculate the Correlation Co-efficient. (Unit-I, Page No. 11, Q17)

X:	2	4	6	8	10	12	14	16	18
Y:	18	16	20	24	22	26	28	32	30

OR

(b) Find out two regression equations from the following data: (Unit-I, Page No. 15, Q22)

X	1	2	3	4	5
Y	2	3	5	4	6

10. (a) Define Index Number. What are its features and uses? (Unit-II, Page No. 30, Q11)

OR

(b) The following are the indices (2007, Base): (Unit-II, Page No. 48, Q31)

Year	2007	2008	2009	2010	2011	2012	2013	2014	2015	2016	2017
Indices	100	120	122	116	120	120	137	136	149	156	137

Shift the base to 2012 and recast the index numbers.

11. (a) What is the least square method? What are the merits and demerits of this method? (Unit-III, Page No. 75, Q18)

OR

(b) Find the straight line tendency for the following data using least square method. (Unit-III, Page No. 79, Q21)

Year	2010	2011	2012	2013	2014	2015
Production	62	83	90	80	90	95

12. (a) What do you mean by probability? Explain the importance of probability. (Unit-IV, Page No. 96, Q12)

OR

(b) A bag contains 4 defective and 6 good Electronic Calculators. Two Calculators are drawn at random one after the other without replacement. Find the probability that

(i) Two are good

(ii) Two are defective and

(iii) One is good and one is defective. (Unit-IV, Page No. 108, Q29)

13. (a) What is normal distribution? Write its any five features. (Unit-V, Page No. 144, Q28)

OR

(b) Ten unbiased coins are tossed simultaneously. Find the probability of obtaining.

(i) Exactly 6 Heads

(ii) Atleast 8 Heads

(iii) No Heads

(iv) Atleast one Head

(v) Not more than 3 Heads and

(vi) Atleast 4 heads. (Unit-V, Page No. 132, Q14)

MODEL PAPER 3

FACULTY OF COMMERCE

B.Com. IV-Semester (CBCS) Examination

BUSINESS STATISTICS-II

Time: 3 Hours Max. Marks: 80

PART - A (5 × 4 = 20 Marks)

[Short Answer Type]

Note: Answer any FIVE of the following questions.

1. Features of Regression Coefficients. (Unit-I, Page No. 2, Q2)
2. Co-efficient of correlation = 0.60, σ_x = 1.5, σ_y = 2.0, x = 10, y = 20, find regression equation y on x. (Unit-I, Page No. 4, Q8)
3. Types of Index Numbers (Unit-II, Page No. 26, Q3)
4. What is Time Series? (Unit-III, Page No. 64, Q1)
5. Given the following equation Ye = 210 + 1.5X. Time origin is 2006. Time unit is one year, shift the origin to 2011. (Unit-III, Page No. 67, Q9)
6. Define Probability. (Unit-IV, Page No. 92, Q1)
7. How many 3 letter words can be formed from the English world "SUCCESS"? (Unit-IV, Page No. 94, Q9)
8. What is the probability of getting 3 heads when a coin is tossed 5 times? (Unit-V, Page No. 126, Q8)

PART - B (5 × 12 = 60 Marks)

(Essay Answer Type)

Note: Answer ALL the questions.

9. (a) Write the relation between correlation and regression. (Unit-I, Page No. 6, Q11)

OR

(b) Given: $\Sigma x = 56$, $\Sigma y = 40$, $\Sigma x^2 = 524$, $\Sigma y^2 = 256$, $\Sigma xy = 364$, N = 8

(i) Find the two Regression equations and

(ii) The Correlation Coefficient. (Unit-I, Page No. 15, Q21)

10. (a) Calculate Fisher's idle index from the following data. (Unit-II, Page No. 40, Q24)

Goods	A	B	C	D	E
P_0(₹)	18	14	16	10	12
Total Cost	1000	600	480	840	720
P_1(₹)	18	16	14	18	20
Total Cost	2,400	960	1050	900	800

OR

(b) Calculate Fisher's Ideal Index Number and test whether it satisfies Time Reversal and Factor Reversal Test for the following data. (Unit-II, Page No. 45, Q28)

Commodity	Base year		Current year	
	Price (₹)	Qty (kg)	Price (₹)	Qty (kg)
A	32	50	30	50
B	30	35	25	40
C	16	55	18	50

11. (a) Find the 4 yearly moving averages from the following data: (Unit-III, Page No. 74, Q16)

Year	2008	2009	2010	2011	2012	2013	2014	2015	2016	2017
Production (in Tonnes)	150	170	196	180	190	216	248	280	300	320

OR

(b) Below are given the figures of production (in thousand quintals) of a sugar factory.

Years	2001	2002	2003	2004	2005	2006	2007
Production	77	88	94	85	91	98	90

(i) Fit a straight line by the least squares method and tabulate the trend values.

(ii) What is the yearly increase in the production of sugar? (Unit-III, Page No. 76, Q19)

12. (a) How many ways are there to paste 2 photos on notice board from a group of 6 photos? (Unit-IV, Page No. 103, Q22)

OR

(b) In a certain college 25% of boys and 10% of girls are studying mathematics. The girls constitute 60% of the students. If a student is selected at random and is found to be studying mathematics, find the probability that the student is a (i) girl (ii) boy. (Unit-IV, Page No. 115, Q36)

13. (a) Six coins are tossed 6400 times. Find the probability to get 6 heads in 2 tosses using Poisson distribution. (Unit-V, Page No. 139, Q22)

OR

(b) A study of past participants indicates that the mean length of time spent on the programme is 500 hours; and that, this normal distribution random variable has a standard deviation of 100 hours. What is the probability that a participant selected at random will required to complete the programme in following cases:

(i) 'More' than 500 hrs

(ii) Between 500 and 650 hrs

(iii) Between 550 and 650 hrs

(iv) Less than 580 hrs

(v) Between 420 and 570 hrs. (Unit-V, Page No. 146, Q30)

Code No. 10173

OSMANIA UNIVERSITY

FACULTY OF COMMERCE

B.COM. IV-Semester (CBCS) Examination

January-2021

BUSINESS STATISTICS -II

Paper Code – BC- 406

(Common Paper for General/Computers and Computer Applications/Advertising/Foreign Trade and Tax Procedures Courses)

Time: 2 Hours | Max. Marks: 80

Part - A (4 × 5 = 20 Marks)

Note: Answer any four questions.

1. If x = 0.85 y and y = 0.89x Find the coefficient of correlation.
2. Define Index Numbers.
3. From the following data, construct on Index Number for 2017 taking 2016 as base as per simple aggregative Method.

Commodity	P	Q	R	S	T
Price 2016(₹)	40	60	85	25	30
Price 2017(₹)	60	90	125	30	40

4. Components of Time Series.
5. From the following data fit a trend line by the method of Semi-Average.

Year:	2012	2013	2014	2015	2016	2017
Output:	20	16	24	30	28	32

6. Explain
   (i) Dependent event and
   (ii) Independent Event.
7. Explain the Axiomatic Approach to probability.
8. Comment on the following,
   For a Binomial Distribution mean = 7 and Variance = 11.

Part - B (4 × 15 = 60 Marks)

Note: Answer any four questions.

9. What is meant by regression? What is the importance and limitations of Analysis?
10. From the following data obtain the two regression equations and calculate the correlation co-efficient.

x	2	4	6	8	10	12	14	16	18
y	18	16	20	24	22	26	28	32	30

Calculate the value of y when x = 6.2

11. The index of 2010 is 100. It rises by 5% in 2011, falls 2% in 2012, rises 4% in 2013, rises 10% in 2014, falls 3% in 2015, and rises 8% in 2016. Find out the indices for the 7 years assuming that all the increases and decreases are the percentages of the respective proceeding years - Also recast the indices shifting base to 2014.

12. From the following data calculate price index according to

(i) Laspeyre,

(ii) Paasche and

(iii) Marshall-Edgeworth methods.

Item	Base year		Current year	
	Price (₹)	Expenditure (₹)	Price (₹)	Expenditure (₹)
A	5	50	8	40
B	7	25	12	30
C	9	10	15	25
D	12	5	20	18

13. Calculate the 3 yearly the 5 yearly moving averages for the following time series.

Year	2006	2007	2008	2009	2010	2011	2012	2013	2014	2015	2016
Production (in quintals)	500	540	550	530	520	560	600	640	620	610	640

14. Obtain the straight line trend equation for the following data by the method of the least square. Tabulate the trend values.

Year	2010	2011	2012	2013	2014	2015	2016
Sale (in '000 units)	140	144	160	152	168	176	180

15. A box contains 8 Red and 5 White balls. Two successive draws of 3 balls are made at random. Find the probability that the first three are white and second three are red.

(i) When there is replacement and

(ii) When there is no replacement.

16. A factory has two machines. Empirical evidence has established that machines I and II produce 30% and 70% of output respectively. It has also been established that 5% and 1% of the output produced by these machines respectively was defective. A defective item is drawn at random. What is the probability that the defective item was produced by either machine I or machine II?

17. 8 Coins are tossed at a time, 256 times. Find the expected frequencies of successes (Getting a Head) and tabulate the results obtained.

18. Fit a poisson distribution to the following data.

x:	0	1	2	3	4
f:	123	59	14	3	1

($e^{-m} = 0.6065$).

SOLUTIONS TO JANUARY-2021, QP

Part - A (4 × 5 = 20 Marks)

Note: Answer any four questions.

Q1. If x = 0.85y and y = 0.89x Find the coefficient of correlation.

Answer :

Given,

$x = 0.85y$ or $bxy = 0.85$

$y = 0.89x$ or $byx = 0.89$

Coefficient of Correlation $(r) = \sqrt{bxy \times byx}$

$= \sqrt{(0.85)\,(0.89)}$

$= \sqrt{0.7565}$

$r = 0.869$ or 0.87.

Q2. Define Index Numbers.

Answer :

For answer refer Unit-II, Page No. 26, Q.No. 1.

Q3. From the following data, construct on Index Number for 2017 taking 2016 as base as per simple aggregative Method.

Commodity	P	Q	R	S	T
Price 2016 (₹)	40	60	85	25	30
Price 2017 (₹)	60	90	125	30	40

Answer :

Computation of Simple Aggregative Method

Commodity	Price in (₹)	
	2016 (P_0)	2017 (P_1)
P	40	60
Q	60	90
R	85	125
S	25	35
T	30	40
Total	$\Sigma P_0 = 240$	$\Sigma P_1 = 350$

Calculation of index number by using simple aggregate method,

$$P_{01} = \frac{\Sigma P_1}{\Sigma P_0} \times 100$$

$$= \frac{350}{240} \times 100$$

$$= 1.4583 \times 100$$

$\therefore P_{01} = 145.83.$

Q4. Components of Time Series.

Answer :

For answer refer Unit-III, Page No. 65, Q.No. 5.

Q5. From the following data fit a trend line by the method of Semi-Average.

Year:	2012	2013	2014	2015	2016	2017
Output:	20	16	24	30	28	32

Answer :

Step-1

The trend mean values for the first 3 years are calculated as follows,

$$= \frac{20+16+24}{3} = \frac{60}{3} = 20$$

The trend mean values for the last 3 years are calculated as follows,

$$= \frac{30+28+32}{3} = \frac{90}{3} = 30$$

Therefore, the Semi-Averages are 20 and 30

Step-2

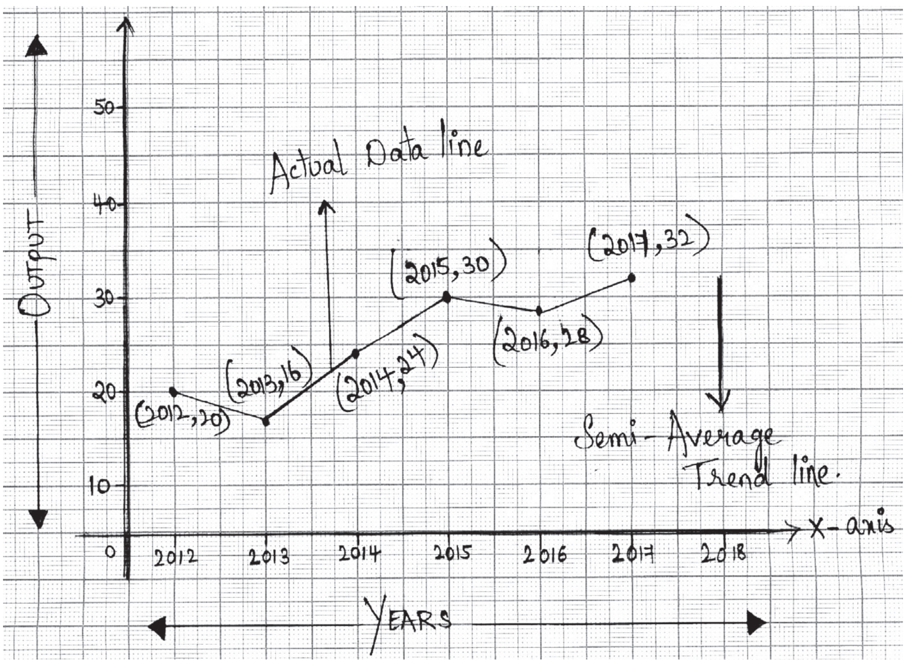
The next step is to plot the semi-averages against the mid-point (middle year) of each time period, thus, it would be year 2013 and 2016 respectively.

Step-3

The plotted points are joined in order to derive the trend line using the semi-average method.

Step-4

The original data and the trend line is plotted on a graph as follows,



Q6. Explain

(i) Dependent event and

(ii) Independent Event.

Answer :

For answer refer Unit-IV, Page No. 96, Q.No. 13, Topic: Dependent Events and Independent Events.

Q7. Explain the Axiomatic Approach to probability.

Answer :

For answer refer Unit-IV, Page No. 105, Q.No. 25, Topic: Axiomatic Approach.

Q8. Comment on the following,

For a Binomial Distribution mean = 7 and Variance = 11.

Answer :

For answer refer Unit-V, Page No. 124, Q.No. 4.

Part - B (4 × 15 = 60 Marks)

Note: Answer any four questions.

Q9. What is meant by regression? What is the importance and limitations of Analysis?

Answer :

Regression

For answer refer Unit-I, Page No. 5, Q.No. 9, Topic: Regression Analysis.

Importance and Limitations of Regression Analysis

For answer refer Unit-I, Page No. 5, Q.No. 10.

Q10. From the following data obtain the two regression equations and calculate the correlation co-efficient.

x	2	4	6	8	10	12	14	16	18
y	18	16	20	24	22	26	28	32	30

Calculate the value of y when x = 6.2.

Answer :

For answer refer Unit-I, Page No. 11, Q.No. 17.

Regression Equation Y on X (When x = 6.2)

Substituting the value of Y when $X = 6.2$ Y will be,

$Y = 0.95x - 14.5$

$Y = 0.95\,(6.2) - 14.5$

$Y = 5.89 - 14.5$

$\boxed{Y = -8.61}$

Q11. The index of 2010 is 100. It rises by 5% in 2011, falls 2% in 2012, rises 4% in 2013, rises 10% in 2014, falls 3% in 2015, and rises 8% in 2016. Find out the indices for the 7 years assuming that all the increases and decreases are the percentages of the respective proceeding years - Also recast the indices shifting base to 2014.

Answer :

Calculation of Index Numbers for Base Year 2010 and Change of Base Year 2014

Years	Old Index Number (Base Year 2010 = 100) $= \frac{100 \pm \%}{100}$ × Previous Year Index number	New Index Number (New Base year 2014 = 116.59) $= \frac{100}{\text{Value of New Base year 2014}}$ × old Index Number of year
2010	100(Given)	$\frac{100}{116.59} \times 100 = 85.77$
2011	$\frac{100+4\%}{100} \times 100 = 104$	$\frac{100}{116.59} \times 104 = 89.20$
2012	$\frac{100-2\%}{100} \times 104 = 101.92$	$\frac{100}{116.59} \times 101.92 = 87.42$
2013	$\frac{100+4\%}{100} \times 101.92 = 105.99$	$\frac{100}{116.59} \times 105.99 = 90.91$
2014	$\frac{100+10\%}{100} \times 105.99 = 116.59$	$\frac{100}{116.59} \times 116.59 = 99.99$
2015	$\frac{100-3\%}{100} \times 116.59 = 113.09$	$\frac{100}{116.59} \times 113.09 = 96.99$
2016	$\frac{100+8\%}{100} \times 113.09 = 122.14$	$\frac{100}{116.59} \times 122.14 = 104.76$

Q12. From the following data calculate price index according to

(i) Laspeyre,

(ii) Paasche and

(iii) Marshall-Edgeworth methods.

Item	Base year		Current year	
	Price (₹)	Expenditure (₹)	Price (₹)	Expenditure (₹)
A	5	50	8	40
B	7	25	12	30
C	9	10	15	25
D	12	5	20	18

Answer :

Note: As price and expenditure are given for base year and current year, divide expenditure of each commodity with their respective price to obtain quantity of base year (i.e., q_0) and quantity of current year (i.e., q_1).

Item	Base year Price (p_0)	Base Year Expenditure ($p_0 q_0$)	Current Year Price (p_1)	Current year Expenditure ($p_1 q_1$)	$q_0 = \frac{p_0 q_0}{p_0}$	$q_1 = \frac{p_1 q_1}{p_1}$	$p_1 q_0$	$p_0 q_1$
A	5	50	8	40	10	5	80	25
B	7	25	12	30	3.6	2.5	43.2	17.5
C	9	10	15	25	1.1	1.7	16.5	15.3
D	12	5	20	18	0.42	0.9	8.4	10.8
		90		**113**			**148.1**	**68.6**

(i) Laspeyre Method

$$P_{01} = \frac{\Sigma p_1 q_0}{\Sigma p_0 q_0} \times 100$$

$$= \frac{148.1}{90} \times 100$$

$$= 164.55$$

(ii) Paasche Method

$$P_{01} = \frac{\Sigma p_1 q_1}{\Sigma p_0 q_1} \times 100$$

$$= \frac{113}{68.6} \times 100$$

$$= 164.72$$

(iii) Marshall - Edgeworth Methods

$$P_{01} = \frac{\Sigma p_1 q_0 + \Sigma p_1 q_1}{\Sigma p_0 q_0 + \Sigma p_0 q_1} \times 100$$

$$= \frac{148.1 + 113}{90 + 68.6} \times 100$$

$$= \frac{261.1}{158.6} \times 100$$

$$= 164.63.$$

Q13. Calculate the 3 yearly the 5 yearly moving averages for the following time series.

Year	2006	2007	2008	2009	2010	2011	2012	2013	2014	2015	2016
Production (in quintals)	500	540	550	530	520	560	600	640	620	610	640

Answer :

Calculation of 3-yearly and 5-yeraly Moving Averages

Years	Production (In Quintals)	3-Yearly Moving Total	3-Yearly Moving Averages	5-Yearly Moving Total	5-Yearly Moving Averages
2006	500	-	-	-	-
2007	540	500 + 540 + 550 = 1,590	$\frac{1590}{3} = 530$	-	-
2008	550	540 + 550 + 530 = 1620	$\frac{1620}{3} = 540$	500 + 540 + 550 + 530 + 520 = 2640	$\frac{2640}{5} = 528$
2009	530	550 + 530 + 520 = 1600	$\frac{1600}{3} = 533.33$	540 + 550 + 530 + 520 + 560 + 2700	$\frac{2700}{5} = 540$
2010	520	530 + 520 + 560 = 1610	$\frac{1610}{3} = 536.66$	550 + 530 + 520 + 560 + 600 = 2760	$\frac{2760}{5} = 552$
2011	560	520 + 560 + 600 = 1680	$\frac{1680}{3} = 560$	530 + 520 + 560 + 600 + 640 = 2850	$\frac{2850}{5} = 570$
2012	600	560 + 600 + 640 = 1800	$\frac{1800}{3} = 600$	520 + 560 + 600 + 640 + 620 = 2940	$\frac{2940}{5} = 588$
2013	640	600 + 640 + 620 = 1860	$\frac{1860}{3} = 620$	560 + 600 + 640 + 620 + 610 = 3030	$\frac{3030}{5} = 606$
2014	620	640 + 620 + 610 = 1870	$\frac{1870}{3} = 623.33$	600 + 640 + 620 + 610 + 640 = 3110	$\frac{3110}{5} = 622$
2015	610	620 + 610 + 640 = 1870	$\frac{1870}{3} = 623.33$	-	-
2016	640	-	-	-	-

Q14. Obtain the straight line trend equation for the following data by the method of the least square. Tabulate the trend values.

Year	2010	2011	2012	2013	2014	2015	2016
Sale (in '000 units)	140	144	160	152	168	176	180

Answer :

Equation for straight line trend is $Y_e = a + bx$. The value of a and b can be attained by solving the following two normal equations,

$\Sigma y = Na + b\Sigma x$ ....(1)

$\Sigma xy = a\Sigma x + b\Sigma x^2$ ...(2)

Now, fitting of straight line trend by the method of least squares.

Year	Sales (in '000 units) y	2013 (x)	x^2	xy	Trend Value Y_e (WN)
2010	140	– 3	9	– 420	139.42
2011	144	– 2	4	– 288	146.28
2012	160	– 1	1	– 160	153.14
2013	152	0	0	0	160
2014	168	1	1	168	166.86
2015	176	2	4	352	173.72
2016	180	3	9	540	180.58
N = 7	**$\Sigma y = 1120$**	**$\Sigma x = 0$**	**$\Sigma y^2 = 28$**	**$\Sigma xy = 192$**	**$\Sigma y_e = 1120$**

Working Notes (WN)

$\Sigma y_e = \text{Na} + b\Sigma x$

$1120 = 7a + 6(0)$

Since the value of $\Sigma x = 0$, $\Sigma y = \text{Na}$

Based on the above table calculation, calculating value of a and b

$$a = \frac{\Sigma y}{N} = \frac{1120}{7} = 160$$

$$b = \frac{\Sigma xy}{\Sigma x^2} = \frac{192}{28} = 6.86$$

Hence, the value of $a = 160$, $b = 6.86$

The equation of straight line trend is,

$Y_e = 160 + 6.86\,x$

2010 $\Rightarrow$ When $x = -3$, $y = 160 + 6.86\,(-3)$

$= 160 - 20.58$

$= 139.42$

2011 $\Rightarrow$ When $x = -2$, $y = 160 + 6.86\,(-2)$

$= 160 - 13.72$

$= 146.28$

2012 $\Rightarrow$ When $x = -1$, $y = 160 + 6.86\ (-1)$

$= 160 - 6.86$

$= 153.14$

2013 $\Rightarrow$ When $x = 0$, $y = 160 + 6.86\ (0)$

$= 160 - 0$

$= 160$

2014 $\Rightarrow$ When $x = 1$, $y = 160 + 6.86\ (1)$

$= 160 + 6.86$

$= 166.86$

2015 $\Rightarrow$ When $x = 2$, $y = 160 + 6.86\ (2)$

$= 160 + 13.72$

$= 173.72$

2016 $\Rightarrow$ When $x = 3$, $y = 160 + 6.86\ (3)$

$= 160 + 20.58$

$= 180.58.$

Q15. A box contains 8 Red and 5 White balls. Two successive draws of 3 balls are made at random. Find the probability that the first three are white and second three are red.

(i) When there is replacement and

(ii) When there is no replacement.

Answer :

(i) When There is Replacement

Given,

Total number of balls in a box = 8 + 5 = 13

3 balls can be drawn from 13 in $^{13}C_3$ Ways.

3 red balls can be drawn form 8 in $^{8}C_3$ Ways

3 White balls can be drawn from 5 in $^{5}C_3$ Ways.

The probability of drawing 3 White balls in the first trial can be find out in the following manner,

$P\ (A\text{n}B) = P(A).P\ (B/A)$

Let A is the event such that the first drawing will give 3 White balls.

Let B is the event such that the Second drawing will given 3 red balls.

Therefore,

$P(A\text{n}B) = P\ (A).P(B/A)$

The probability of 3 white balls at first trial is,

$$P(A) = \frac{^{5}C_3}{^{13}C_3} = \frac{5}{143}$$

The probability of 3 red balls at the second trial is,

$$P(B/A) = \frac{8c_3}{13c_3} = \frac{28}{143}$$

$$\therefore P(AnB) = \frac{5}{143} \times \frac{28}{143} = \frac{140}{20,449}$$

$$\Rightarrow 0.007$$

Working Notes (WN)

$$nc_r = \frac{n!}{r!(n-r)!}$$

1. $${}^5C_3 = \frac{5!}{3!(5-3)!}$$

$$= \frac{5\times4\times3\times2\times1}{3\times2\times1(2)!}$$

$$= \frac{5\times4\times\cancel{3\times2\times1}}{\cancel{3\times2\times1}(2\times1)}$$

$$= \frac{5\times4}{2\times1}$$

$$= \frac{20}{2} = 10$$

2. $${}^{13}C_3 = \frac{13!}{3!(13-3)!}$$

$$= \frac{13\times12\times11\times\cancel{10\times9\times8\times7\times6\times5\times4\times3\times2\times1}}{3\times2\times1(\cancel{10\times9\times8\times7\times6\times5\times4\times3\times2\times1})}$$

$$= \frac{13\times\cancel{12}^{4}\times11}{\cancel{3}\times2\times1}$$

$$= \frac{13\times4\times11}{1\times2\times1}$$

$$= \frac{572}{2} = 286$$

$$\therefore \frac{8c_3}{13c_3} = \frac{\cancel{10}^{5}}{\cancel{289}_{143}} = \frac{5}{143}$$

(ii) When There is No Replacement

The probability of drawing 3 white balls in the first trial is,

$$P(AnB) = P(A) \cdot P(B/A)$$

$$P(A) = \frac{{}^5C_3}{{}^{13}C_3} = \frac{5}{143}$$

When the white balls are drawn and are not replaced, the box contains 2 white balls and 8 red balls.

$\therefore$ At, the second trial 3 balls can be drawn from 10 in ${}^{10}C_3$ Ways and 3 red balls can be drawn from 8 in ${}^{8}C_3$ Ways .

The probability of 3 red balls in the second trial,

$$P(B/A) = \frac{{}^7C_3}{{}^{10}C_3} = \frac{7}{24}$$

$$\therefore P(AnB) = \frac{5}{143} \times \frac{7}{24} = \frac{7}{429}$$

$$= 0.0102.$$

Working Notes (WN)

$$nc_r = \frac{n!}{r!(n-r)!}$$

1. $$^7C_3 = \frac{7!}{3!(7-3)!}$$

$$= \frac{7\times6\times5\times4\times3\times2\times1}{3\times2\times1\,(4)}$$

$$= \frac{7\times6\times5\times\cancel{4\times3\times2\times1}}{3\times2\times1\,(\cancel{4\times3\times2\times1})}$$

$$= \frac{\cancel{210}^{35}}{\cancel{6}_1} = 35$$

2. $$^{10}C_3 = \frac{10!}{3!(10-3)!}$$

$$= \frac{10\times9\times8\times7\times6\times5\times4\times3\times2\times1}{3\times2\times1\,(7)!}$$

$$= \frac{10\times9\times8\times\cancel{7\times6\times5\times4\times3\times2\times1}}{3\times2\times1\,(\cancel{7\times6\times5\times4\times3\times2\times1})}$$

$$= \frac{10\times\cancel{9}^3\times\cancel{8}^4}{\cancel{3}_1\times\cancel{2}_1\times1} = \frac{720}{6}$$

$$= 120$$

$$\frac{^7C_3}{10C_3} = \frac{35}{120} = \frac{7}{24}$$

Q16. A factory has two machines. Empirical evidence has established that machines I and II produce 30% and 70% of output respectively. It has also been established that 5% and 1% of the output produced by these machines respectively was defective. A defective item is drawn at random. What is the probability that the defective item was produced by either machine I or machine II?

Answer :

Let 'A_1' be the event of drawing an item produced by machine 1.

While, 'A_2' is the event of drawing an item produced by machine 2.

Let, 'B' be the event of drawing defective item which is produced by either of the machines. From the additional information given,

$P(A_1)$ is the probability of getting an item produced by machine 1.

$P(A_2)$ is the probability of getting an item produced by machine 2.

$P(B/A_1)$ is the probability of getting defective item produced by machine 1.

$P(B/A_2)$ is the probability of getting defective item produced by machine 2.

From the given data,

$P(A_1) = 0.3$ (30%) $P(B/A_1) = 5\% = \frac{5}{100} = 0.05$

$P(A_2) = 0.7$ (70%) $P(B/A_2) = 1\% = \frac{1}{100} = 0.01$

Computation of Posterior Probabilities

Events	Priori Probability P(A$_i$)	Conditional Probability P(B/A$_i$)	Joint Probability $P(A_i \cap B)$	Posterior Probability P(A$_i$/B) = $\frac{P(A_i \cap B)}{\Sigma(A_i \cap B)}$
A_1	$P(A_1) = 0.3$	$P(B/A_1) = 0.05$	$0.3\times0.05 = 0.015$	$P(A_1/B) = \frac{0.015}{0.022} = 0.682$
A_2	$P(A_2) = 0.7$	$P(B/A_2) = 0.01$	$0.7\times0.01 = 0.007$	$P(A_2/B) = \frac{0.007}{0.022} = 0.318$
Total	**1.00**	**0.06**	$\Sigma P(A_i \cap B) = 0.022$	**1.000**

Therefore, the posterior probability obtained after calculation are as follows,

If a defective item is drawn at random,

(i) The probability of defective item produced by machine 1, $P(A_1/B)$ = 0.682 or = 68%

(ii) The probability of defective item produced by machine 2, $P(A_2/B)$ = 0.318 = 31.8% or $\simeq$ 38

Note:

Always the sum of priori probability as well as posterior probabilities is equal to 1 or 100.

Q17. 8 Coins are tossed at a time, 256 times. Find the expected frequencies of successes (Getting a Head) and tabulate the results obtained.

Answer :

Given that,

$n = 8$

$N = 256$

The Probability of getting a head $(p) = \frac{1}{2}$

The Probability of getting a tail $(q) = \frac{1}{2}$

The Probability of Success r times in n trials is given by ${}^nC_r\, q^{n-r}\, p^r$

$\therefore\ P(r) = {}^nC_r\, q^{n-r}\, p^r$

$$= {}^8C_r \left(\frac{1}{2}\right)^{8-r} \left(\frac{1}{2}\right)^r$$

$$= {}^8C_r \left(\frac{1}{2}\right)^r$$

The frequencies of 0,1,2,3,...,8 Successes are as follows,

Success	N×P(r)	Expected Frequency
0	$256\left(\frac{1}{256}\times 8c_0\right)$	1
1	$256\left(\frac{1}{256}\times 8c_1\right)$	8
2	$256\left(\frac{1}{256}\times 8c_2\right)$	28
3	$256\left(\frac{1}{256}\times 8c_3\right)$	56
4	$256\left(\frac{1}{256}\times 8c_4\right)$	70
5	$256\left(\frac{1}{256}\times 8c_5\right)$	56
6	$256\left(\frac{1}{256}\times 8c_6\right)$	28
7	$256\left(\frac{1}{256}\times 8c_7\right)$	8
8	$256\left(\frac{1}{256}\times 8c_8\right)$	1
	Total	**256**

Working Notes

Sample Calculation of N × P (r)

(i) $${}^{n}C_{r} = \frac{n!}{r!\,(n-r)!}$$

$${}^{8}C_{3} = \frac{8!}{3!\,(8-3)!}$$

$${}^{8}C_{3} = \frac{8!}{3!\,(5)!}$$

$${}^{8}C_{3} = \frac{{}^{4}\cancel{8}\times 7\times {}^{2}\cancel{6}\times \cancel{5\times 4\times 3\times 2\times 1}}{\cancel{3}_{1}\times \cancel{2}_{1}\times 1\,(\cancel{5\times 4\times 3\times 2\times 1})}$$

$$= \frac{4\times 7\times 2}{1\times 1\times 1} = \frac{4\times 7\times 2}{1} = 56$$

$$= 256\left(\frac{1}{256}\times 8c_{3}\right)$$

$$= 256\left(\frac{1}{256}\times 56\right)$$

$$= 256\,(0.21875)$$

$$= 56$$

(ii) $${}^{8}C_{6} = \frac{8!}{6!\,(8-6)!}$$

$$= \frac{8\times 7\times \cancel{6\times 5\times 4\times 3\times 2\times 1}}{\cancel{6\times 5\times 4\times 3\times 2\times 1}\,(2)!}$$

$$= \frac{\cancel{8}^{4}\times 7}{{}^{1}\cancel{2}\times 1} = 28$$

$$= 256\left(\frac{1}{256}\times 28\right) = 28$$

Q18. Fit a poisson distribution to the following data.

x:	0	1	2	3	4
f:	123	59	14	3	1

(e^{-m} = 0.6065).

Answer :

Step-1

Calculate the values of ‘λ’ and Probability of zero Occurrence.

X	f	fx
0	123	0
1	59	59
2	14	28
3	3	9
4	1	4
	$\Sigma f = 200$	**$\Sigma fx = 100$**

Mean of poisson distribution is given

$$\lambda = \frac{\Sigma fx}{\Sigma f}$$

$$\lambda = \frac{100}{200} = 0.50$$

$$\therefore\ \lambda = 0.50$$

Step-2

Calculate all the probabilities by using recurrence relation,

$P(0) = 0.6065$ ($\therefore$ Given e^{-m} (or) $e^{-\lambda}$ 0.6065)

$$P(1) = \frac{P(0)\times\lambda}{1} \Rightarrow 0.6065\times0.5 = 0.3032$$

$$P(2) = \frac{P(1)\times\lambda}{2} = \frac{0.3032\times0.5}{2} = 0.0758$$

$$P(3) = \frac{P(2)\times\lambda}{3} = \frac{0.0758\times0.5}{3} = 0.013$$

$$P(4) = \frac{P(3)\times\lambda}{4} = \frac{0.013\times0.5}{4} = 0.002$$

Step-3

Multiply each term of probability with total frequency (Σf) to obtain the values of expected frequencies.

Computation of Expected Frequencies

X	P(x)	f(x) = N.P (x) = 200 P(x)
0	$P(0) = 0.6065$	$f(0) = 200 \times 0.6065 = 121.3 \cong 121$
1	$P(1) = 0.3032$	$f(1) = 200 \times 0.3032 = 60.64 \cong 61$
2	$P(2) = 0.08$	$f(2) = 200 \times 0.0758 = 15.16 \cong 15$
3	$P(3) = 0.013$	$f(3) = 200 \times 0.013 = 2.6 \cong 3$
4	$P(4) = 0.001$	$f(4) = 200 \times 0.001\ 0.2 \cong 0$
	Total	**200**

$\therefore$ The theoretically fitted poisson distribution is as follows,

x	0	1	2	3	4
y	121	61	15	3	0

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OSMANIA UNIVERSITY

FACULTY OF COMMERCE

B.Com. Year IV-Semester (CBCS) Examination

May/June-2018

BUSINESS STATISTICS-II

(Paper Code – BC - 406)

(Common Paper for General / Computers / Computer Applications / Advertising / Foreign Trade and Tax Procedure Courses)

Time: 3 Hours Max. Marks: 80

Part - A (5 × 4 = 20 Marks)

Note: Answer any Five of the following questions not exceeding 20 lines each.

1. If $\gamma = 0.6$, $\sigma x = 1.5$ and $\sigma y = 2$, find the b_{xy} and b_{yx}. (Unit-I, Page No. 4, Q7)

2. Importance of Index Numbers. (Unit-II, Page No. 26, Q2)

3. From the following data calculate a price index based on price relatives method using Arithmetic Mean: (Unit-II, Page No. 28, Q8)

Commodity	A	B	C	D	E	F
Price 2015 (₹)	45	60	20	50	85	120
Price 2016 (₹)	55	70	30	75	90	130

4. What are the uses of Time Series? (Unit-III, Page No. 64, Q2)

5. Explain (i) Mutually Exclusive Events and (ii) Not-Mutually Exclusive Events. (Unit-IV, Page No. 92, Q2)

6. When two dice are thrown, find the probability that the sum of the numbers is either 10 or 11. (Unit-IV, Page No. 93, Q6)

7. 6 coins are tossed at a time, what is the probability of obtaining 4 or more heads? (Unit-V, Page No. 125, Q7)

8. Properties of Normal Distribution. (Unit-V, Page No. 125, Q5)

Part - B (5 × 12 = 60 Marks)

Note: Answer all the questions in not exceeding 4 pages each.

9. (a) Define Regression and what are the differences between correlation and regression. (Unit-I, Page No. 7, Q13)

OR

(b) Given:

$\Sigma x = 56$, $\Sigma y = 40$, $\Sigma x^2 = 524$, $\Sigma y^2 = 256$, $\Sigma xy = 364$, $N = 8$

(i) Find the two Regression equations and

(ii) The Correlation Coefficient. (Unit-I, Page No. 15, Q21)

10. (a) The following are the indices (2007, Base):

Year	2007	2008	2009	2010	2011	2012	2013	2014	2015	2016	2017
Indices	100	120	122	116	120	120	137	136	149	156	137

Shift the base to 2012 and recast the index numbers. (Unit-II, Page No. 48, Q31)

OR

(b) From the following data calculate Price Index Number by using (i) Paasche's Method and (ii) Marshal Edgeworth Method. (Unit-II, Page No. 41, Q25)

Item	Base Year		Current Year	
	Price (₹)	Expenditure (₹)	Price (₹)	Expenditure (₹)
P	6	300	10	560
Q	2	200	2	240
R	4	240	6	360
S	10	300	12	288
T	3	120	8	240

11. (a) Find the 4 yearly moving averages from the following data: (Unit-III, Page No. 74, Q16)

Year	2008	2009	2010	2011	2012	2013	2014	2015	2016	2017
Production (in Tonnes)	150	170	196	180	190	216	248	280	300	320

OR

(b) Production figure of a Textile Industry are as follows, (Unit-III, Page No. 77, Q20)

Year	2011	2012	2013	2014	2015	2016	2017
Production (in '000 units)	12	10	14	11	13	15	16

For the above data,

(i) Determine the straight line equation under the Least Square Method.

(ii) Find the Trend Values and show the trend line on a graph paper.

12. (a) From 30 tickets marked with first 30 numerals, 1 ticket is drawn at random. It is then replaced and a second draw is made. Find the probability that in the first draw it is multiple of 5 or 7 and in second draw it is a multiple of 3 or 7. (Unit-IV, Page No. 109, Q30)

OR

(b) In a bolt factory, the Machines P, Q and R manufacture respectively 25%, 35% and 40% of the total of their outputs 5,4,2 percents respectively are defective bolts. A bolt is drawn at random from the product, and is known to be defective. What are the probabilities that it was manufactured by the machines P, Q and R. (Unit-IV, Page No. 113, Q35)

13. (a) Five coins are tossed 3,200 times, find the frequencies of the distribution of heads and tails; and tabulate the results and also calculate Mean and Standard Deviation of fitted distribution. (Unit-V, Page No. 134, Q15)

OR

(b) A study of past participants indicates that the mean length of time spent on the programme is 500 hours; and that, this normal distribution random variable has a standard deviation of 100 hours. What is the probability that a participant selected at random will required to complete the programme in following cases:

(i) 'More' than 500 hrs

(ii) Between 500 and 650 hours

(iii) Between 550 and 650 hours

(iv) Less than 580 hours

(v) Between 420 and 570 hours. (Unit-V, Page No. 146, Q30)

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May/June-2019

BUSINESS STATISTICS-II

(Paper Code – BC - 406)

(Common Paper for General / Computers / Computer Applications / Advertising / Foreign Trade and Tax Procedure Courses)

Time: 3 Hours | Max. Marks: 80

Part - A (5 × 4 = 20 Marks)

[Short Answer Type]

Note: Answer any five of the following questions not exceeding 20 lines each.

1. If r = 0.8; σ_x = 2.5, = σ_y = 3.5, find bxy and byx, **(Unit-I, Page No. 4, Q6)**
2. Types of Index Numbers. **(Unit-II, Page No. 26, Q3)**
3. Calculate Index number by Average Price Relative Method by using Arithmetic Mean. **(Unit-II, Page No. 28, Q9)**

Commodity	P	Q	R	S	T
Price 2017	2	6	10	5	12
Price 2018	4	8	15	5	8

4. Utility of Time Series Analysis. **(Unit-III, Page No. 64, Q2)**
5. Fit a trend line to the following data by the Freehand Method. **(Unit-III, Page No. 65, Q6)**

Year	2009	2010	2011	2012	2013	2014	2015	2016	2017
Sales (₹)	19	22	24	20	23	25	23	26	25

6. Explain:
   (i) Mutually exclusive events and
   (ii) Dependent events. **(Unit-IV, Page No. 92, Q2)**
7. One card is drawn at random from a pack of 52 cards. What is the probability that it is either a king or a queen?
8. Comment on the following: **(Unit-III, Page No. 93, Q7)**
   For a Binomial Distribution Mean = 7 and Variance = 11. **(Unit-V, Page No. 124, Q4)**

PART – B (5 × 12 = 60 Marks)

[Essay Answer Type]

Note: Answer all the questions in not exceeding 4 pages each by using internal choice.

9. (a) From the following data obtain the two regression equations and calculate the Correlation Co-efficient. **(Unit-I, Page No. 11, Q17)**

X:	2	4	6	8	10	12	14	16	18
Y:	18	16	20	24	22	26	28	32	30

OR

(b) Following are the marks in Statistics and English in an Annual Examination. (Unit-I, Page No. 12, Q18)

	Statistics (X)	English (Y)
Mean	40	50
Standard Derivation	10	16
Co-efficient Correlation	0.5	

(i) Estimate the score of English, when the score in Statistics is 50.

(ii) Estimate the score of Statistics, when the score in English is 30.

10. (a) Define Index Number. What are its features and uses? (Unit-II, Page No. 30, Q11)

OR

(b) Compute Price Index Number by using: (Unit-II, Page No. 39, Q22)

(i) Paasches and

(ii) Marshal and Edgeworth methods.

Commodity	Base Year		Current Year	
	Price	Quantity	Price	Quantity
P	5	100	6	150
Q	4	80	5	100
R	2	60	5	72
S	12	30	9	33

11. (a) From the following data, calculate trend values using Four Yearly Moving Averages. (Unit-III, Page No. 73, Q15)

Year	2009	2010	2011	2012	2013	2014	2015	2016	2017
Production (in tons)	506	620	1036	673	588	696	1116	738	663

OR

(b) Fit a straight line by the Least Square Method and tabulate the trend values for the above data. (Unit-III, Page No. 76, Q19)

Year	2011	2012	2013	2014	2015	2016	2017
Production (in tons)	77	88	94	85	91	98	90

12. (a) A bag contains 4 defective and 6 good Electronic Calculators. Two Calculators are drawn at random one after the other without replacement. Find the probability that (Unit-IV, Page No. 109, Q29)

(i) Two are good

(ii) Two are defective and

(iii) One is good and one is defective.

OR

(b) A company has two Plants for manufacturing Scooters. Plant I manufactures 80% of the Scooters and Plant II manufactures 20%. At the Plant I 85% Scooters are rated to be of standard quality and at Plant II 65% Scooters are rated to be of standard quality. One Scooter was selected at random. What is the probability that (Unit-V, Page No. 112, Q34)

(i) It is manufactured by Plant I

(ii) It is manufactured by Plant II – which is of standard quality.

13. (a) Ten unbiased coins are tossed simultaneously. Find the probability of obtaining. (Unit-V, Page No. 132, Q14)

(i) Exactly 6 Heads

(ii) Atleast 8 Heads

(iii) No Heads

(iv) Atleast one Head

(v) Not more than 3 Heads and

(vi) Atleast 4 heads.

OR

(b) Fit a Poisson distribution to the following data: (Unit-V, Page No. 138, Q21)

X	0	1	2	3	4
Y	211	90	19	5	0

$(e^{-m} = 0.6443)$

UNIT 1

Regression

SYLLABUS

Introduction - Linear and Non-Linear Regression – Correlation Vs. Regression - Lines of Regression - Derivation of Line of Regression of Y on X - Line of Regression of X on Y - Using Regression Lines for Prediction.

LEARNING OBJECTIVES

- ✓ *Concept, Types and Applications of Regression.*
- ✓ *Linear and Non-Linear Regression.*
- ✓ *Differences between Correlation and Regression.*
- ✓ *Lines of Regression.*
- ✓ *Derivation of Line of Regression of Y on X.*
- ✓ *Derivation of Line of Regression of X on Y.*
- ✓ *Use of Regression Lines for Prediction.*

INTRODUCTION

Regression analysis attempts to establish the 'nature of the relationship' between variables that is, to study the functional relationship between the variables and thereby provide a mechanism for prediction or forecasting.

Linear regression is a form of regression which is used for modelling the relationship between scalar variables like 'X' and 'Y'. Under linear regression, linear functions are used to model the data and the unknown parameters of models are estimated from the data. Hence, these models are known as linear models.

In the non-linear regression the explained variable (dependent variable) changes at varying rate with a given change in the explaining variable (independent variable). It is also known as Curvilinear Regression. Under Non-Linear Regression, the observational data are modelled by a function i.e., a non-linear blend of model parameters and depends on one or more independent variable.

In order to predict the value of variable 'Y' (dependent variable) for a given value of variable 'X' (independent variable), the equation of regression line is preferable.

PART-A SHORT QUESTIONS AND ANSWERS

Q1. Define Regression Analysis.

Answer : Model Paper-I, Q1

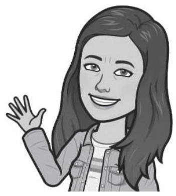
According to Ya-Lun Chou, "Regression analysis attempts to establish the 'nature of the relationship' between variables that is, to study the functional relationship between the variables and thereby provide a mechanism for prediction or forecasting".

According to Morris Hamburg, "the term 'regression analysis' refers to the methods by which estimates are made of the values of variable from a knowledge of the values of one or more other variables and to the measurement of the errors involved in this estimation process".

By the above two definitions, we can define Regression Analysis as a measure of the average relationship between two or more variables in terms of original units of the data.

Q2. Features of Regression Coefficients.

Answer : (Model Paper-III, Q1 | May/June-19, Q1(MGU))



Keep an eye on me

The properties/features of regression coefficients are as follows,

1. **Deviations**

When deviations taken from actual mean,

"$r.\frac{\sigma_y}{\sigma_x}$" is denoted by "$b_{xy}$" and is given by,

$$b_{xy} = \frac{\sum(x-\bar{x})(y-\bar{y})}{\sum(x-\bar{x})^2}$$

"$r.\frac{\sigma_x}{\sigma_y}$" is denoted by "$b_{yx}$" and is given by,

$$b_{yx} = \frac{\sum(x-\bar{x})(y-\bar{y})}{\sum(y-\bar{y})^2}$$

When deviations taken from assumed mean,

$$b_{yx} = \frac{N\Sigma dxdy - (\Sigma dx)(\Sigma dy)}{N\Sigma dx^2 - (\Sigma dx)^2}$$

$$b_{xy} = \frac{N\Sigma dxdy - (\Sigma dx)(\Sigma dy)}{N\Sigma dy^2 - (\Sigma dy)^2}$$

2. **Least Square Line**

The least square regression line always passes through $(\bar{x}, \bar{y})$.

3. **Same Sign**

Both regression coefficients have the same sign.

4. **Sign of 'r'**

Correlation coefficient has the same sign as that of regression coefficients.

5. **Independent of origin**

Regression coefficients are independent of change of origin but not of scale.

6. **r is Geometric Mean**

Correlation coefficient (r) is the geometric mean of two regression coefficients.

$$r = \pm\sqrt{b_{yx} \times b_{xy}}$$

$$r = \sqrt{r.\frac{\sigma_x}{\sigma_y} \times \frac{\sigma_y}{\sigma_x}}$$

Q3. What are the types of regression?

Answer :

The various types of regression are as follows,

1. Simple Regression

The regression analysis confined to the study of only two variables at a time is termed as simple regression.

2. Multiple Regression

The regression analysis for studying more than two variables at a time is termed as multiple regression.

3. Linear Regression

If the regression curve is a straight line, the regression is termed as linear regression. The equation of such a curve is the equation of a straight line i.e., first degree equation in variables x and y.

4. Non-linear Regression

If the curve of the regression is not a straight line, the regression is termed as curved or non-linear regression. The regression equation will be a functional relation between variables x and y involving terms in x and y of degree more than one.

Q4. Define the principle of least squares and standard error of estimate.

Answer :

Principle of Least Squares

The principle of least squares consists of minimizing the sum of the squares of the residuals or the errors of estimates, i.e., the deviations between the given observed values of the variable and their corresponding estimated values as given by the line of best fit. Lines of regression uses the principle of least squares to give the best fit line for estimating the value of one variable given the value of another variable.

Standard Error of Estimate

The standard error of estimate is a measure of the accuracy of predictions. The estimates obtained by using the regression equations may not be perfect. A measure of precision of these estimate is given by the standard error of the estimate. Standard deviation gives us measure of dispersion of the observations about the mean of the distribution whereas standard error of estimate gives us a measure of the observations about the line of regression.

Formula of Standard Error of Estimate is as follows,

$$\text{Standard error of estimate } (S_{yx}) = \sqrt{\frac{\Sigma(Y - Y_c)^2}{N}}$$

Q5. Write three limitations of regression analysis.

Answer : May/June-18, Q1(a) (KU)

Some of the limitations of Regression Analysis are as follows,

1. Regression analysis assumes that linear relationship exists among the related variables. But in the area of social sciences, linear relationship may not exist among the related variables.

2. When regression analysis is used to evaluate the value of dependent variable based on independent variable, it is assumed that the static conditions of relationship exist between them. These statistic conditions do not exist in social sciences, so this assumption minimizes the use of regression analysis in social science.

3. The value of dependent variable can be evaluated based on independent variable by using regression analysis but only upto some limits. If the circumstances go beyond the limits, then results would be inaccurate.

Q6. If r = 0.8; σ_x = 2.5, σ_y = 3.5, find b_{xy} and b_{yx}.

Answer : May/June-19, Q1 (OU)

Given that,

$$r = 0.8, \sigma_x = 2.5, \sigma_y = 3.5$$

(i)

$$b_{xy} = r\,\frac{\sigma_x}{\sigma_y}$$
$$= 0.8\left(\frac{2.5}{3.5}\right)$$
$$= 0.8 \times 0.714$$
$$= 0.571$$

(ii)

$$b_{yx} = r\,\frac{\sigma_y}{\sigma_x}$$
$$= 0.8\left(\frac{3.5}{2.5}\right)$$
$$= 0.8 \times 1.4$$
$$= 1.12$$

Q7. If γ = 0.6, σ_x = 1.5 and σ_y = 2, find the b_{xy} and b_{yx}.

Answer : (Model Paper-II, Q1 | May/June-18, Q1 (OU))

Given that,

r = 0.6, σ_x = 1.5 and σ_y = 2

$$b_{xy} = r.\frac{\sigma_x}{\sigma_y}$$
$$b_{xy} = 0.6 \times \frac{1.5}{2}$$
$$= 0.45$$
$$b_{yx} = r.\frac{\sigma_y}{\sigma_x}$$
$$b_{yx} = 0.6 \times \frac{2}{1.5}$$
$$= 0.8$$

Q8. Co-efficient of correlation = 0.60, σ_x = 1.5, σ_y = 2.0, x = 10, y = 20, find regression equation y on x.

Answer : (Model Paper-III, Q2 | May/June-18, Q1(g) (KU))

$$b_{yx} = r \times \frac{\sigma_y}{\sigma_x}$$
$$b_{yx} = 0.60 \times \frac{2}{1.5}$$
$$b_{yx} = 0.8$$

Regression Equation of y on x

$$y - \bar{y} = b_{yx}(x - \bar{x})$$
$$y - 20 = 0.8(x - 10)$$
$$y - 20 = 0.8x - 8$$
$$y = 0.8x - 8 + 20$$
$$y = 0.8x + 12$$

PART-B ESSAY QUESTIONS AND ANSWERS

1.1 REGRESSION – INTRODUCTION

Q9. What is regression analysis? Explain regression variables and types of regression.

Answer :

Regression Analysis

Regression analysis is a technique which expresses the relationship between two or more variables in equation form to estimate the value of variable based on the given value of another variable.

Regression takes its name from studies made by Sir Francis Galton. He compared the heights of persons to the heights of their parents. His major conclusion was that the children of unusually tall persons tend to be shorter than their parents while children of short parents tend to be shorter than their parents.

In a sense, the successive generations of off-springs from tall persons "regress" downward toward the height of the population. While the reverse is true originally for short families. But the distribution of heights for the total population, continues to have the same variability from generation to generation.

Regression Variables

The regression variables are as follows,

1. Independent Variable (Regressor or Predictor or Explanatory)

The variable which influences the values of the other variable or which is used for predicting the value of the other variable is called independent variable.

2. Dependent Variable (Regressed or Explained Variable)

The variable whose value is influenced or is to be predicted is called dependent variable. Regression test generates lines of regression of the two variables which helps in estimating the values. Lines of regression of y on x is the line which gives the best estimate for the value of y for any specified value of x. Similarly, line of regression of x on y is the line which gives the best estimate for the value of x for any specified value of y.

Types of Regression

The various types of regression are as follows,

1. Simple Regression

The regression analysis confined to the study of only two variables at a time is termed as simple regression.

2. Multiple Regression

The regression analysis for studying more than two variables at a time is termed as multiple regression.

3. Linear Regression

If the regression curve is a straight line, the regression is termed as linear regression. The equation of such a curve is the equation of a straight line i.e., first degree equation in variables x and y.

4. Nonlinear Regression

If the curve of the regression is not a straight line, the regression is termed as curved or non-linear regression. The regression equation will be a functional relation between variables x and y involving terms in x and y of degree more than one.

Q10. What are the applications/utility of regression test? State the limitations of regression analysis.

Answer :

Applications/Utility/Importance of Regression Test

Regression lines or equations are useful in the predictions of values of one variable for a specified value of the other variable. Some of the applications/utility of regression test are as follows,

1. For pharmaceutical firms which are interested in studying the effect of new drugs in patients, regression test helps in such predictions.
2. When price and demand are related, we can estimate or predict the future demand for a specified price.
3. When crop yield depends on the amount of rainfall, then regression test can predict crop yield for a particular amount of rainfall.
4. If advertising expenditure and sales are related, then regression analysis helps in estimating the advertising expenditure for a required amount of sales or sales expected for a particular advertising expenditure.
5. When capital employed and profits earned are related, the test can be used to predict profits for a specified amount of capital invested.

Limitations of Regression Analysis

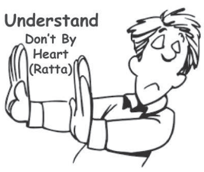
Some of the limitations of Regression Analysis are as follows,

1. Regression analysis assumes that linear relationship exists among the related variables. But in the area of social sciences, linear relationship may not exist among the related variables.
2. When regression analysis is used to evaluate the value of dependent variable based on independent variable, it is assumed that the static conditions of relationship exist between them. These statistic conditions do not exist in social sciences, so this assumption minimizes the use of regression analysis in social science.
3. The value of dependent variable can be evaluated based on independent variable by using regression analysis but only upto some limits. If the circumstances go beyond the limits, then results would be inaccurate.

Q11. Write the relation between correlation and regression.

Answer : (Model Paper-III, Q9(a) | May/June-18, Q2(a) (KU))

The following points highlight the relation between correlation and regression.



1. Both correlation and regression analysis are key statistical tools for studying the functional relationship between two (or more) variables. They both help in determining the nature and strength of relationship between the variables.
2. They both help the decision makers in prediction and reduction of uncertainty.
3. They both are interdependent.
4. They both can be demonstrated with the help of graphs which are known as scatterplots.
5. When the correlation is negative or positive, the regression slope i.e., line within the graph will also be negative or positive.
6. They both involve straight line relationship.
7. The scores on both variables represent continuous scores.
8. In both analysis, the error is measured in a very similar way.
9. The correlation coefficient 'r' is linked to the coefficient of determination 'R^2' in the regression analysis.
10. The correlation coefficient 'r' takes same sign as taken by 'b' in regression analysis.

1.1.1 Linear and Non-Linear Regression

Q12. What do you mean by linear and non-linear regression? Distinguish between them.

Answer :

Linear Regression

Linear Regression is a form of regression which is used for modeling the relationship between scalar variables like '*X*' and '*Y*'. Under linear regression, linear functions are used to model the data and the unknown parameters of models are estimated from the data. Hence, these models are known as Linear Models.

Linear Models commonly refers to those models, where the conditional mean of variable '*Y*' for a given value of variable '*X*' will be an affine function of *X*. A linear regression may also refer to a model, where median or other quantile of the conditional distribution of '*Y*' for a given value of '*X*' is termed as linear function of *X*. Similar, to all types of regression analysis, linear regression also aims on the conditional probability distribution of '*Y*' for a given '*X*', instead of joint probability distribution of '*Y*' and '*X*'.

Non-Linear Regression

In the Non-Linear Regression the explained variable (dependent variable) changes at varying rate with a given change in the explaining variable (independent variable). It is also known as Curvilinear Regression. Under non-linear regression, the observational data are modeled by a function i.e., a non-linear blend of model parameters and depends on one or more independent variable. Method of successive approximations are used for fitting the data. The data in non-linear regression contains of error free independent variable 'X' and its relatively observed dependent variable 'Y'.

Example

The output of rice increases rapidly with the application of the initial dose of fertilizer; there after it increases at a falling rate. The relationship in such case, when shown on graph will yield a ‘curve’.

Differences between Linear and Non-linear Regression

The differences between Linear and Non-Linear Regression are as follows,

S.No.	Basis	Linear Regression	Non-Linear Regression
1.	Meaning	It is a form of regression which is used for modelling the relationship between a scalar variable ‘X’ and ‘Y’.	It is a type of regression, where the observational data are modeled by a function i.e., a nonlinear blend of model parameters.
2.	Curve	If the regression curve is a straight line, then the regression is termed as linear regression.	If the curve of the regression is not a straight line, then the regression is termed as curved or nonlinear regression.
3.	Model form	In this, the parameters are considered as linear combinations.	In this, the parameter are considered as functions.
4.	Solution	In this, the solution for parameters is represented as closed form.	In this, it is necessary for parameters to be solved repeatedly by using optimization algorithms.
5.	Uniqueness	The solution under linear regression is unique.	The Sum of the Squared Errors (SSE) may not be appear as unique.
6.	Parameters estimation	In case of un-correlated errors, estimation of parameters are unbiased.	Incase of un-correlated errors, estimation of parameters are usually biased.
7.	Equation	The equation of regression curve is the equation of a straight line i.e., first degree equation in variables X and Y.	The regression equation will be functional relation between variables X and Y involving terms in x and y of degree more than one.

1.2 CORRELATION VS REGRESSION

Q13. Define regression and what are the differences between correlation and regression.

Answer : (Model Paper-I, Q9(a) | May/June-18, Q9(a) (OU))

Regression

According to M.M. Blair, “Regression is a mathematical measure of the average relationship between two or more variables in terms of the original units of the data”.

Differences between Correlation and Regression

The comparison between Correlation and Regression are as follows,

S.No	Basis	Correlation	Regression
1.	Meaning	It means the relationship between two or more variables which vary in sympathy so that the movements in one variable tend to be accompanied by the corresponding movements in others.	It means stepping back or returning to average value and is a mathematical measure expressing the average relationship between the variables.
2.	Nature	It is a measure of the ‘degree and direction’ of relationship between the variables.	It studies ‘nature’ of relationship between the variables.
3.	Cause and Effect Relationship	It does not indicate the cause and effect relationship between the variables.	It clearly indicates the cause and effect relationship between the variables.
4.	Variables	It cannot say which variable is the dependent variable and which is the independent variable.	The variable corresponding to cause is taken as independent variable and the variable corresponding to effect is taken as dependent variable.

5.	Relative and Absolute Measures	It coefficient is a relative measure of the linear relationship.	Regression coefficients are absolute measures indicating the change in the value of one variable for a unit change in the value of the other variable.
6.	Prediction	It cannot be used for predicting or estimating value.	It is very helpful in predicting and estimating value of one variable given the value of another variable.
7.	Coefficients	Correlation coefficients are symmetric i.e., $r_{yx} = r_{xy}$.	Regression coefficients are asymmetric i.e., $b_{xy} \neq b_{yx}$.
8.	Range/Limit	The range of r is +1 to – 1.	The range of b_{xy} and b_{yx} is not restricted.
9.	Relationship	Its coefficient can be calculated from regression coefficients.	Its coefficients cannot be directly compared from correlation coefficient.
10.	Drawbacks	There may be non-sense correlation between variables due to chance.	There is no such thing in regression.

1.3 LINES OF REGRESSION - DERIVATION OF LINE OF REGRESSION OF Y ON X - LINE OF REGRESSION OF X ON Y

Q14. What do you mean by lines of regression? Derive the equation of lines of regression.

Answer :

Lines of Regression

In a bi-variate distribution, if the variables are related then the points when plotted in the scatter diagram will lie near a straight line which is called as line of regression and the regression is said to be Linear Regression. If points lie on some non-linear curve then the regression is said to be Curvi-Linear Regression.

The lines of regression gives the best estimate to the value of a variable for any given value of another variable. Thus it is the line of "best fit" which is obtained by using the principles of least squares.

Derivation of Lines of Regression of Y on X

Let X and Y be two variables. Also assume that Y is dependent variable and X is independent variable and the bi-variate distribution is (x_i, y_i) for $i = 1, 2, ... n$.

Let the line of regression of Y on X be,

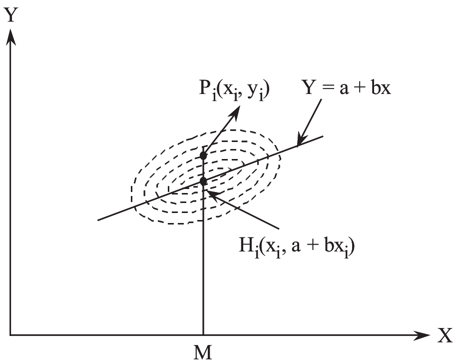
$$Y = a + bX \qquad ... (1)$$

Where a and b are constants. We need to find the values of a and b such that this line is the line of "best fit".

Let $P_i(x_i, y_i)$ be any point in the scatter diagram. Now draw a line P_iM perpendicular to the x-axis so that it meets the line in equation (1). The intersecting point of the two lines is H_i whose coordinates are $(x_i, a + bx_i)$.

Thus, $P_iH_i = P_iM - H_iM = y_i - (a + bx_i)$

This is called the error of estimate or the residual for y_i.



According to the principle of least squares we have to determine a and b such that E is minimum.

$$E = \sum_{i=1}^{n} P_i H_i^2 = \sum_{i=1}^{n} (y_i - a - bx_i)^2$$

The partial derivative of E with respect to a and b is,

$$\frac{\partial E}{\partial a} = -2\sum_{i=1}^{n}(y_i - a - bx_i)$$

$$\Rightarrow \quad \frac{0}{2} = -\left[\left(\sum_{i=1}^{n} y_i\right) - na - b\sum_{i=1}^{n} x_i\right]$$

$$\Rightarrow \quad 0 = -\sum_{i=1}^{n} y_i + na + b\sum_{i=1}^{n} x_i$$

$$\sum_{i=1}^{n} y_i = na + b\sum_{i=1}^{n} x_i \qquad \text{... (2)}$$

$$\Rightarrow \quad \frac{\partial E}{\partial b} = -2\sum_{i=1}^{n} x_i(y_i - a - bx_i)$$

$$\Rightarrow \quad \frac{0}{2} = -\left[\sum_{i=1}^{n} x_i y_i - a\sum_{i=1}^{n} x_i - b\sum_{i=1}^{n} x_i^2\right]$$

$$\Rightarrow \quad 0 = -\sum_{i=1}^{n} x_i y_i + a\sum_{i=1}^{n} x_i + b\sum_{i=1}^{n} x_i^2$$

$$\Rightarrow \quad \sum_{i=1}^{n} x_i y_i = a\sum_{i=1}^{n} x_i + b\sum_{i=1}^{n} x_i^2 \qquad \text{... (3)}$$

The equations (2) and (3) are known as the normal equations for estimating a and b.

Divide equation (2) by n, we get,

$$\bar{y} = a + b\bar{x} \qquad \text{... (4)}$$

This equation indicates that the regression line of Y on X passes through the point $(\bar{x}, \bar{y})$.

Now,

$$\mu_{11} = \text{Cov}(X, Y)$$

$$\mu_{11} = \frac{1}{n}\sum_{i=1}^{n} x_i y_i - \bar{x}\,\bar{y}$$

$$\Rightarrow \quad \frac{1}{n}\sum_{i=1}^{n} x_i y_i = \mu_{11} + \bar{x}\,\bar{y} \qquad \text{... (5)}$$

And $\quad \sigma_x^2 = \frac{1}{n}\sum_{i=1}^{n} x_i^2 - \bar{x}^2$

$$\Rightarrow \quad \frac{1}{n}\sum_{i=1}^{n} x_i^2 = \sigma_x^2 + \bar{x}^2$$

Divide equation (3) by n, we get,

$$\frac{1}{n}\sum_{i=1}^{n} x_i y_i = a.\frac{1}{n}\sum_{i=1}^{n} x_i + b.\frac{1}{n}\sum_{i=1}^{n} x_i^2 \qquad \text{... (6)}$$

Equating equations (5) and (6), we get,

$$\mu_{11} + \bar{x}\,\bar{y} = a\bar{x} + b\left(\frac{1}{n}\sum_{i=1}^{n} x_i^2\right)\left[\because \frac{1}{n}\sum_{i=1}^{n} x_i^2 = {\sigma_x}^2 + \bar{x}^2\right]$$

$$\mu_{11} + \bar{x}\,\bar{y} = a\bar{x} + b(\sigma_x^2 + \bar{x}^2) \qquad \text{... (7)}$$

Multiply equation (4) by $\bar{x}$ and then subtract from equation (7), we get,

$$\Rightarrow \quad \mu_{11} + \bar{x}\,\bar{y} - \bar{x}\,\bar{y} = a\bar{x} + b(\sigma_x^2 + \bar{x}^2) - a\bar{x} - b\bar{x}^2$$

$$\Rightarrow \quad \mu_{11} = b\sigma_x^2$$

$$\Rightarrow \quad b = \frac{\mu_{11}}{\sigma_x^2}$$

Since, the slope of the regression line of Y on X (i.e., equation (1)) is b and the line passes through the point $(\bar{x}, \bar{y})$ and, the equation for the line is,

$$Y - \bar{y} = b(X - \bar{x})$$

$$Y - \bar{y} = \frac{\mu_{11}}{\sigma_x^2}(X - \bar{x})$$

$$Y - \bar{y} = r\frac{\sigma_Y}{\sigma_X}(X - \bar{x}) \qquad \text{... (8)}$$

The equation (8) is the regression line of Y on X.

Derivation of Line of Regression Line of X on Y

Let the line of regression of X on Y is,

$$X = a + bY$$

The equation of the line of regression of X on Y can be obtained by following the same procedure used to obtain the equation of the line of regression of Y on X or by just interchanging the variables X and Y.

Thus, the equation of the line of regression of X on Y is,

$$X - \bar{x} = \frac{\mu_{11}}{\sigma_Y^2}(Y - \bar{y})$$

$$X - \bar{x} = r\frac{\sigma_X}{\sigma_Y}(Y - \bar{y})$$

1.3.1 Regression Coefficient

Q15. Explain briefly about Regression Coefficient.

Answer :

Regression Coefficient

The regression coefficient X on Y measures the change in X corresponding to a unit change in Y and the regression coefficient of Y on X measures the change in Y corresponding to a unit change in X,

$r.\frac{\sigma_y}{\sigma_x}$ *and* $r.\frac{\sigma_x}{\sigma_y}$ are known as coefficient of regression.

Properties of Regression Coefficient

The properties/features of regression coefficients are as follows,

1. Deviations

When deviations taken from actual mean,

"$r.\frac{\sigma_y}{\sigma_x}$" is denoted by "$b_{xy}$" and is given by,

$$b_{xy} = \frac{\sum(x-\bar{x})(y-\bar{y})}{\sum(x-\bar{x})^2}$$

"$r.\frac{\sigma_x}{\sigma_y}$" is denoted by "$b_{yx}$" and is given by,

$$b_{yx} = \frac{\sum(x-\bar{x})(y-\bar{y})}{\sum(y-\bar{y})^2}$$

When deviations taken from assumed mean,

$$b_{yx} = \frac{N\Sigma dxdy - (\Sigma dx)(\Sigma dy)}{N\Sigma dx^2 - (\Sigma dx)^2}$$

$$b_{xy} = \frac{N\Sigma dxdy - (\Sigma dx)(\Sigma dy)}{N\Sigma dy^2 - (\Sigma dy)^2}$$

2. Least Square Line

The least square regression line always passes through $(\bar{x}, \bar{y})$.

3. Same Sign

Both regression coefficients have the same sign.

4. Sign of 'r'

Correlation coefficient has the same sign as that of regression coefficients.

5. Value

If one value of regression coefficient is greater than unity, then the other must be less than unity, since r |>|1 (But both can be less than unity).

6. Average Value

Average value of the two regression coefficients will be greater than correlation coefficient.

7. Independent of origin

Regression coefficients are independent of change of origin but not of scale.

8. r is Geometric Mean

Correlation coefficient (r) is the geometric mean of two regression coefficients.

$$r = \pm\sqrt{b_{yx} \times b_{xy}}$$

$$= \pm\sqrt{r.\frac{\sigma_x}{\sigma_y} \times r.\frac{\sigma_y}{\sigma_x}}$$

1.4 USING REGRESSION LINES FOR PREDICTION

Q16. Explain briefly about the regression lines used for prediction.

Answer :

In order to predict the value of variable 'Y' (dependent variable) for a given value of variable 'X' (independent variable), the equation of regression line is preferable.

Example

The predicted value of variable 'Y' (Y_i) for a given variable 'X' (X_i) will be represented as,

$$Y_i = a + bX_i$$

Here, a and b are obtained from sample data and are least squares estimates derived from the normal equations.

Thus, it is important to use regression equations carefully for prediction and estimation. It is preferable to use regression equation for estimation when it properly fits data. Therefore, goodness is required to be tested before using lines of regression.

Following are some of the important points which need to be considered while using regression lines for prediction,

1. The significance of observed sample correlation coefficient $r = r(X, Y)$ is need to be tested. The lines of regression for estimation and prediction can be used when the value of 'r' is significant.
2. Linear model is not a good fit when the 'r' value is not significant. Therefore, lines of regression cannot be used in this case.
3. If 'r' is significant and the linear regression is a good fit for the given data, then it is preferable to use line of regression for estimating 'Y' for given 'X'.
4. Linear regression model should not be used for predicting 'Y' corresponding to far distant value of X because lot of changes may occur in the pattern of relationship between these two variables. Therefore, the predicted value of Y for distant value of X may not be worthy.
5. It is useful to make predictions for linear regression model when sample data is drawn from the population.

PROBLEMS ON REGRESSION

Q17. From the following data obtain the two regression equations and calculate the Correlation Co-efficient.

X:	2	4	6	8	10	12	14	16	18
Y:	18	16	20	24	22	26	28	32	30

Solution : (Model Paper-II, Q9(a) | May/June-19, Q9(a) (OU))

Obtaining Two Regression Equations

1. X on Y

$X - \overline{X} = b_{xy}(Y - \overline{Y})$

2. Y on X

$Y - \overline{Y} = b_{yx}(X - \overline{X})$

$\therefore \overline{X} = \frac{\Sigma x}{n} = \frac{90}{9} = 10$

$\therefore \overline{Y} = \frac{\Sigma y}{n} = \frac{216}{9} = 24$

X	Y	$X-\overline{X}$ (X – 10) x	$Y-\overline{Y}$ (Y – 24) y	x^2	y^2	xy
2	18	– 8	– 6	64	36	48
4	16	– 6	– 8	36	64	48
6	20	– 4	– 4	16	16	16
8	24	– 2	0	4	0	0
10	22	0	– 2	0	4	0
12	26	2	2	4	4	4
14	28	4	4	16	16	16
16	32	6	8	36	64	48
18	30	8	6	64	36	48
$\Sigma x = 90$	$\Sigma y = 216$	$\Sigma x = 0$	$\Sigma y = 0$	$\Sigma x^2 = 240$	$\Sigma y^2 = 240$	$\Sigma xy = 228$

$b_{xy} = \frac{\Sigma xy}{\Sigma y^2} = \frac{228}{240} = 0.95$

$b_{yx} = \frac{\Sigma xy}{\Sigma x^2} = \frac{228}{240} = 0.95$

Regression Equation X on Y

$X - \overline{X} = b_{xy}(Y - \overline{Y})$

$X - 10 = 0.95\,(Y - 24)$

$X - 10 = 0.95Y - 22.8$

$X = 0.95Y - 22.8 + 10$

$\boxed{\therefore X = 0.95y - 12.8}$

Regression Equation Y on X

$$Y - \overline{Y} = b_{yx}(X - \overline{X})$$

$$Y - 24 = 0.95\,(X - 10)$$

$$Y - 24 = 0.95\,X - 9.5$$

$$Y = 0.95X - 9.5 + 24$$

$$\boxed{\therefore Y = 0.95x - 14.5}$$

Correlation Coefficient

$$r = \pm\sqrt{b_{yx} \times b_{xy}}$$

$$= \pm\sqrt{0.95 \times 0.95}$$

$$= \pm\sqrt{0.9025}$$

$$= \pm 0.95$$

Since, both regression coefficients are positive, ‘r’ must be positive.

Hence, correlation coefficient = + 0.95

Q18. Following are the marks in Statistics and English in an Annual Examination.

	Statistics (X)	English (Y)
Mean	40	50
Standard Derivation	10	16
Coefficient Correlation	0.5	

(i) Estimate the score of English, when the score in Statistics is 50.

(ii) Estimate the score of Statistics, when the score in English is 30.

Solution : (Model Paper-I, Q9(b) | May/June-19, Q9(b) (OU))

Given that,

$\overline{x} = 40$, $\overline{y} = 50$, $\sigma_x = 10$, $\sigma_y = 16$, $r = 0.5$


Solve Me with Concentration

(i) Regression Equation of X on Y

$$X - \overline{X} = r\left[\frac{\sigma_x}{\sigma_y}\right](Y - \overline{Y})$$

$$\Rightarrow \quad X - 40 = (0.5)\left[\frac{10}{16}\right](Y - 50)$$

$$\Rightarrow \quad X - 40 = (0.5) \times (0.625)\,(Y - 50)$$

$$\Rightarrow \quad X - 40 = (0.3125)\,(Y - 50)$$

$$\Rightarrow \quad X - 40 = 0.3125\,Y - 15.625$$

$$\Rightarrow \quad X = 0.3125\,Y - 15.625 + 40$$

$$\Rightarrow \quad X = 0.3125\,Y + 24.375$$

When $Y = 30$

$$\Rightarrow \quad X = 0.3125\,(30) + 24.375 = 9.375 + 24.375$$

$$= 33.75 \simeq 34$$

When score in English is 30, score of statistics will be 34.

(ii) Regression Equation of Y on X

$$Y - \overline{Y} = r\left[\frac{\sigma_y}{\sigma_x}\right](X - \overline{X})$$

$$\Rightarrow \quad Y - 50 = (0.5)\left[\frac{16}{10}\right](X - 40)$$

$$\Rightarrow \quad Y - 50 = 0.5\,(1.6)\,(X - 40)$$

$$\Rightarrow \quad Y - 50 = (0.8)\,(X - 40)$$

$$\Rightarrow \quad Y - 50 = 0.8\,X - 32$$

$$\Rightarrow \quad Y = 0.8\,X - 32 + 50$$

$$\Rightarrow \quad Y = 0.8\,X + 18$$

When $X = 50$

$$\Rightarrow \quad Y = 0.8\,(50) + 18 = 40 + 18$$

$$= 58$$

When score in statistics is 50, score of English will be 58.

Q19. Calculate two regression equations from the following data.

X	4	6	8	13	13	12	14	15
Y	6	8	12	10	14	12	15	20

Solution : May/June-19, Q6(a) (MGU)

Calculations for Regression Equation

X	dx (X – 11)	dx²	Y	dy (Y – 12)	dy²	dxdy
4	– 7	49	6	– 6	36	42
6	– 5	25	8	– 4	16	20
8	– 3	9	12	0	0	0
13	2	4	10	– 2	4	– 4
13	2	4	14	2	4	4
12	1	1	12	0	0	0
14	3	9	15	3	9	9
15	4	16	20	8	64	32
Σx = 85	**Σdx = – 3**	**Σdx² = 117**	**Σy = 97**	**Σdy = 1**	**Σy² = 133**	**Σdxdy = 103**

$$\overline{X} = \frac{\Sigma x}{n} = \frac{85}{8} = 10.625$$

$$\overline{Y} = \frac{\Sigma y}{n} = \frac{97}{8} = 12.125$$

Since mean values $\overline{X}$ and $\overline{Y}$ are non-integer values, deviations are taken from assumed mean as 11 and 12 respectively.

Regression Coefficient of Y on X

$$b_{yx} = \frac{n\Sigma dxdy - (\Sigma dx)(\Sigma dy)}{n\Sigma dx^2 - (\Sigma dx)^2} = \frac{8 \times 103 - (-3)(1)}{8 \times 117 - (-3)^2}$$

$$= \frac{824 + 3}{936 - 9} = \frac{827}{927}$$

$$= 0.8921$$

Regression Equation of Y on X

$$y - \overline{Y} = b_{yx}(X - \overline{X})$$

$$Y - 12.125 = 0.8921(X - 10.625)$$

$$Y - 12.125 = 0.8921X - 9.4786$$

$$Y = 0.8921X - 9.4786 + 12.125$$

$$Y = 0.8921X + 2.6464$$

Regression Coefficient of X on Y

$$b_{xy} = \frac{n\Sigma dxdy - (\Sigma dx)(\Sigma dy)}{n\Sigma dy^2 - (\Sigma dy)^2}$$

$$= \frac{8 \times 103 - (-3)(1)}{8 \times 133 - (1)^2} = \frac{824 + 3}{1064 - 1} = \frac{827}{1063}$$

$$= 0.778$$

Regression Equation of X on Y

$$X - \overline{X} = b_{xy}(Y - \overline{Y})$$

$$X - 10.625 = 0.778\,(Y - 12.125)$$

$$X - 10.625 = 0.778Y - 9.4332$$

$$X = 0.778\,Y - 9.4332 + 10.625$$

$$X = 0.778Y + 1.1918$$

Q20. Calculate two regression equations from the following data.

X	2	3	4	5	6
Y	3	2	5	6	4

Solution : May/June-19, Q6(b) (MGU)

$$\overline{X} = \frac{\Sigma x}{N} = \frac{20}{5} = 4$$

$$\overline{Y} = \frac{\Sigma y}{N} = \frac{20}{5} = 4$$

$$\therefore \quad \overline{X} = 4,\ \overline{Y} = 4$$

X	Y	(X – 4)	(Y – 4)	x^2	y^2	xy
2	3	– 2	– 1	4	1	2
3	2	– 1	– 2	1	4	2
4	5	0	1	0	1	0
5	6	1	2	1	4	2
6	4	2	0	4	0	0
Σx = 20	**Σy = 20**	**Σx = 0**	**Σy = 0**	**$\Sigma x^2 = 10$**	**$\Sigma y^2 = 10$**	**Σxy = 6**

1. Regression Equation of X on Y

$$X - \overline{X} = b_{xy}(Y - \overline{Y})$$

$$b_{xy} = \frac{\Sigma xy}{\Sigma y^2} = \frac{6}{10} = 0.6$$

$$X - 4 = 0.6\,(Y - 4)$$

$$X - 4 = 0.6\,Y - 2.4$$

$$X = 0.6\,Y - 2.4 + -4$$

$$\therefore \quad X = 0.6\,Y + 1.6$$

2. Regression Equation of Y on X

$$Y - \overline{Y} = b_{yx}(X - \overline{X})$$

$$b_{yx} = \frac{\Sigma xy}{\Sigma x^2} = \frac{6}{10} = 0.6$$

$$Y - 4 = 0.6\,(X - 4)$$

$$X - 4 = 0.6\,X - 2.4$$

$$Y = 0.6\,X - 2.4 + 4$$

$$\therefore \quad Y = 0.6\,X + 1.6$$

Q21. Given:

$\Sigma x = 56$, $\Sigma y = 40$, $\Sigma x^2 = 524$, $\Sigma y^2 = 256$, $\Sigma xy = 364$, $N = 8$

(i) Find the two Regression equations and

(ii) The Correlation Coefficient.

Solution : (Model Paper-III, Q9(b) | May/June-18, Q9(b) (OU))

Given ,

$\Sigma x = 56$, $\Sigma y = 40$, $\Sigma x^2 = 524$, $\Sigma y^2 = 256$, $\Sigma xy = 364$, $N = 8$

(i) Two Regression Equations

Regression Equation of X On Y

$$x - \overline{x} = b_{xy}\,(y - \overline{y})$$

$$\overline{x} = \frac{\Sigma x}{n} = \frac{56}{8} = 7$$

$$b_{xy} = \frac{n\Sigma xy - (\Sigma x)(\Sigma y)}{n\Sigma y^2 - (\Sigma y)^2}$$

$$= \frac{8 \times 364 - (56)(40)}{8 \times 256 - (40)^2}$$

$$= \frac{2,912 - 2,240}{2,048 - 1,600}$$

$$= \frac{672}{448}$$

$$= 1.5$$

$$\therefore x - \overline{x} = b_{xy}\,(y - \overline{y})$$

$$x - 7 = 1.5\,(y - 5)$$

$$x - 7 = 1.5y - 7.5$$

$$x = 1.5y - 7.5 + 7$$

$$x = 1.5y - 0.5$$

Regression Equation of Y On X

$$y - \overline{y} = b_{yx}\,(x - \overline{x})$$

$$\overline{y} = \frac{\Sigma y}{n} = \frac{40}{8} = 5$$

$$b_{yx} = \frac{n\Sigma xy - (\Sigma x)(\Sigma y)}{n\Sigma x^2 - (\Sigma x)^2}$$

$$= \frac{8 \times 364 - (56)(40)}{8 \times 524 - (56)^2}$$

$$= \frac{2,912 - 2,240}{4,192 - 3,136}$$

$$= \frac{672}{1056}$$

$$= 0.6364$$

$$\therefore \; y - \overline{y} = b_{yx}\,(x - \overline{x})$$

$$y - 5 = 0.6364\,(x - 7)$$

$$y - 5 = 0.6364x - 4.4548$$

$$y = 0.6364x - 4.4548 + 5$$

$$y = 0.6364x + 0.5452$$

(ii) Correlation Coefficient

$$r = \pm\sqrt{b_{yx} \times b_{xy}}$$

$$= \pm\sqrt{0.6364 \times 1.5}$$

$$= \pm\sqrt{0.9546}$$

$$= \pm\,0.98$$

Q22. Find out two regression equations from the following data:

X	1	2	3	4	5
Y	2	3	5	4	6

Solution : (Model Paper-II, Q9(b) | May/June-18, Q2(b) (KU))

Obtaining Two Regression Equations

1. X on Y

$$X - \overline{X} = b_{xy}\,(y - \overline{y})$$

2. Y on X

$$y - \overline{y} = b_{yx}\,(x - \overline{x})$$

$$\overline{X} = \frac{\Sigma X}{N} = \frac{15}{5} = 3$$

$$\overline{Y} = \frac{\Sigma Y}{N} = \frac{20}{5} = 4$$

X	Y	$X - \bar{X}$ (X – 3) x	$Y - \bar{Y}$ (Y – 4) y	x^2	y^2	xy
1	2	– 2	– 2	4	4	4
2	3	– 1	– 1	1	1	1
3	5	0	1	0	1	0
4	4	1	0	1	0	0
5	6	2	2	4	4	4
$\Sigma X = 15$	$\Sigma Y = 20$	$\Sigma x = 0$	$\Sigma y = 0$	$\Sigma x^2 = 10$	$\Sigma y^2 = 10$	$\Sigma xy = 9$

$$b_{xy} = \frac{\Sigma xy}{\Sigma y^2} = \frac{9}{10} = 0.9$$

$$b_{yx} = \frac{\Sigma xy}{\Sigma x^2} = \frac{9}{10} = 0.9$$

1. Regression Equation X on Y

$$X - \bar{X} = b_{xy}(Y - \bar{Y})$$

$$X - 3 = 0.9(Y - 4)$$

$$X - 3 = 0.9Y - 3.6$$

$$X = 0.9Y - 3.6 + 3$$

$$\therefore X = 0.9\,Y - 0.6$$

2. Regression Equation Y on X

$$Y - \bar{Y} = b_{yx}(X - \bar{X})$$

$$Y - 4 = 0.9(X - 3)$$

$$Y - 4 = 0.9X - 2.7$$

$$Y = 0.9X - 2.7 + 4$$

$$\therefore Y = 0.9\,X + 1.3$$

Q23. Following is the distribution of students according to their heights and weights.

Heights (in inches)	Weight in (Lbs) 90-100	100-110	110-120	120-130
50-55	4	7	5	2
55-60	6	10	7	4
60-65	6	12	10	7
65-70	3	8	6	3

Calculate,

(i) The two coefficients of regression

(ii) Obtain the two regression equations.

Solution :

Let height be 'x' and weight be 'y'

(i) The Two Coefficients of Regression

(a) Regression coefficient of x on y

$$b_{xy} = \frac{N.\Sigma fdxdy - \Sigma fdx\ \Sigma fdy}{N.\Sigma fd^2y - (\Sigma fdy)^2} \times \frac{i_x}{i_y}$$

(b) Regression coefficient of y on x

$$b_{yx} = \frac{N.\Sigma fdxdy - \Sigma fdx\ \Sigma fdy}{N.\Sigma fd^2x - (\Sigma fdx)^2} \times \frac{i_y}{i_x}$$

Where,

i_y = Difference between class intervals of weight

= 100 – 90 = 10

i_x = Difference between class intervals of height

= 55 – 50 = 5

Calculations of Coefficient Regression

Height (x)	Mid - value	M.V.	Weight (y)				f	fdx	$f_2$dx	fdxdy
			90-100 95	100-110 105	110-120 115	120-130 125				
		dx \ dy	–1	0	1	2				
50-55	52.5	–1	4 4	7 0	5 –5	2 –4	18	–18	18	–5
55-60	57.5	0	6 0	10 0	7 0	4 0	27	0	0	0
60-65	62.5	1	6 –6	12 0	10 10	7 14	35	35	35	18
65-70	67.5	2	3 –6	8 0	6 12	3 12	20	40	80	18
		f	19	37	28	16	N=100	57	133	31
		fdy	–19	0	28	32	41			
		fdy²	19	0	28	64	111			
		fdxdy	–8	0	17	22	31			

Here, $\sum fdxdy = 31, \sum fdx = 57, \sum fdy = 41, N = 100$

$\sum fdx^2 = 133$ and $\sum fdy^2 = 111$

(a) Regression Coefficient of x on y

$$b_{xy} = \frac{N.\Sigma fdxdy - \Sigma fdx.\Sigma fdy}{N.\Sigma fdy^2 - (\Sigma fdy)^2} \times \frac{i_x}{i_y} = \frac{100 \times 31 - (57)(41)}{100 \times 111 - (41)^2} \times \frac{5}{10}$$

$$= \frac{3100 - 2337}{11100 - 1681} \times \frac{1}{2} = \frac{763}{9419} \times \frac{1}{2}$$

$$= 0.0810 \times 0.5 = 0.0405 \simeq 0.041$$

(b) Regression Coefficient y on x

$$b_{xy} = \frac{N.\Sigma fdxdy - \Sigma fdx.\Sigma fdy}{N.\Sigma fdx^2\ (\Sigma fdx)^2} \times \frac{i_y}{i_x}$$

$$= \frac{100 \times 31 - (57)(41)}{100 \times 133 - (57)^2} \times \frac{10}{5}$$

$$= \frac{763}{13300 - 3249} \times \frac{10}{5} = \frac{763}{10051} \times 2$$

$$= 0.0759 \times 2 = 0.1518 \simeq 0.152$$

(ii) Two Regression Equations

(a) Regression Equation of x on y

$$x - \bar{x} = b_{xy}(y - \bar{y})$$
$$x - 60.35 = 0.041(y - 109.1)$$
$$x - 60.35 = 0.041y - 4.473$$
$$x = 0.041y - 4.473 + 60.35$$
$$x = 0.041y + 55.88$$

(b) Regression Equation of y on x

$$y - \bar{y} = b_{yx}(x - \bar{x})$$
$$y - 109.1 = 0.152(x - 60.35)$$
$$y = 0.152\,x - 9.17 + 109.1$$
$$y = 0.152\,x + 99.93$$

Working Notes

Calculation of $\bar{x}$ and $\bar{y}$

$$\bar{x} = A + \frac{\Sigma fdx}{N} \times i_x$$

Where,

A = Assumption

$$\bar{x} = 57.5 + \frac{57}{100} \times 5$$
$$= 57.5 + 2.85$$
$$\bar{x} = 60.35$$

$$\bar{y} = A + \frac{\Sigma fdy}{N} \times i_y$$
$$\bar{y} = 105 + \frac{41}{100} \times 10$$
$$\bar{y} = 105 + 4.1 = 109.1$$

Q24. You are given the following information about advertisement expenditure and sales:

	Adv. Exp (X) (₹ Crores)	Sales (Y) (₹ Crores)
Mean	20	120
S.D	5	25
Correlation coefficient	0.8	

1. **Calculate two regression equations.**
2. **Find likely sales when Adv. Expenses is ₹ 25 Crores.**
3. **What should be the Adv. Budget if the company wants to attain sales target of ₹ 150 crores**

Solution : Oct./Nov.-13, Q12(b) (OU)

Let, the variable x represent advertisement expenses and y represent sales (in ₹ crores). Then we have,

$$\bar{x} = 20,\ \bar{y} = 120,\ \sigma_x = 5,\ \sigma_y = 25,\ r = 0.8$$

1. Two Regression Equations

(i) Regression Equation of X on Y

$$x - \bar{x} = r\left[\frac{\sigma_x}{\sigma_y}\right](y - \bar{y})$$
$$\Rightarrow \quad x - 20 = 0.8\left[\frac{5}{25}\right](y - 120)$$
$$\Rightarrow \quad x - 20 = 0.16\,(y - 120)$$
$$\Rightarrow \quad x - 20 = 0.16\,y - 19.2$$
$$\Rightarrow \quad x = 0.16y - 19.2 + 20$$
$$\Rightarrow \quad x = 0.16\,y + 0.8$$

(ii) Regression Equation of Y on X

$$y - \bar{y} = r\left[\frac{\sigma_y}{\sigma_x}\right](x - \bar{x})$$
$$\Rightarrow \quad y - 120 = 0.8\left[\frac{25}{5}\right](x - 20)$$
$$\Rightarrow \quad y - 120 = 4(x - 20)$$
$$y - 120 = 4x - 80$$
$$\Rightarrow \quad y = 4x - 80 + 120$$
$$\Rightarrow \quad y = 4x + 40$$

∴ The two equations are,

$$x = 0.16y + 0.8$$
$$y = 4x + 40$$

2. For proposed advertisement expenditure of ₹ 25 crores, sales,

$$y = 4x + 40$$
$$= 4(25) + 40$$
$$= ₹\ 140 \text{ crore}$$

3. To achieve the sales target of ₹ 150 crores the company should have the advertising budget,

$$x = 0.16y + 0.8$$
$$= 0.16\,(150) + 0.8$$
$$= ₹\ 24.8 \text{ crore.}$$

EXERCISE AND PRACTICE QUESTIONS

SHORT QUESTIONS

THEORY

Q1. Define Regression Analysis. [Refer, Q1]

Q2. Features of Regression Coefficients. [Refer, Q2] May/June-19, Q1(MGU)

Q3. Define the principle of least squares and standard error of estimate. [Refer, Q4]

Q4. Write three limitations of regression analysis. [Refer, Q5] May/June-18, Q1(a) (KU)

PROBLEMS

Q5. If r = 0.6; $\sigma_x = 5$, $\sigma_y = 7$, find b_{xy} and b_{yx}. [Refer Similar, Q6]

(Ans: $b_{xy} = 0.4284$, $b_{yx} = 0.84$)

Q6. If $\gamma = 0.8$, $\sigma_x = 3$ and $\sigma_y = 4$, find the b_{xy} and b_{yx}. [Refer Similar, Q7]

(Ans: $b_{xy} = 0.6$, $b_{yx} = 1.067$)

Q7. Co-efficient of correlation = 0.60, $\sigma_x = 3$, $\sigma_y = 4$, x = 10, y = 20 find regression equation y on x. [Refer Similar, Q8]

(Ans: $b_{yx} = 0.79$ or 0.8 ; Y on X; Y = 0.8 X + 12)

Q8. Given the two regression coefficient X on Y = +0.542 and Y on X = + 0.905. Calculate the coefficient of correlation between X and Y. [Refer Similar, Q21]

(Ans: +0.70)

Q9. If regression equation of X on Y = 0.268 and of Y on X = 0.5, find coefficient of correlation. [Refer Similar, Q21]

(Ans: +0.259)

Q10. If the correlation coefficient (r) = 0.86 and regression coefficient of X on Y = 1.2, find the regression coefficient of Y on X. [Refer Similar, Q23]

(Ans: 0.6163)

ESSAY QUESTIONS

THEORY

Q11. What is regression analysis? Explain regression variables and types of regression. [Refer, Q9]

Q12. What are the applications/utility of regression test? State the limitations of regression analysis. [Refer, Q10]

Q13. Write the relation between correlation and regression. [Refer, Q11] May/June-18, Q2(a) (KU)

Q14. What do you mean by linear and nonlinear regression? Distinguish between them. [Refer, Q12]

Q15. Define regression and what are the differences between correlation and regression. [Refer, Q13]

May/June-18, Q9(a) (OU)

PROBLEMS

Q16. From the following data obtain the regression equation of x on y and also that of y on x. [Refer Similar, Q17]

x	6	2	10	4	8
y	9	11	5	8	7

(Ans: x = 1.3y + 16.4; y = – 0.65 x + 11.9)

Q17. From the following data obtain the two regression equations and calculate the correlation co-efficient. [Refer Similar, Q17]

x	1	2	3	4	5	6	7	8	9
y	9	8	10	12	11	13	14	16	15

Estimate the value of y which should correspond on an average to X = 6.2

(Ans: y = 13.14)

Q18. From the data given below find out,

(a) Co-efficient of correlation between the ages of husbands and the ages of wives.

(b) The two regression equations.

(c) The expected age of husband when wife's age is 14.

(d) The expected age of wife when husband's age is 35. [Refer Similar, Q17, Q18]

Age of Husband (in years)	22	23	23	24	26	27	27	28	30	30
Age of Wife (in years)	18	20	21	20	21	22	23	24	25	26

(Ans: x = 1.1y + 1.8, y = 0.8 x + 1.2, r = 0.94, x = 17.2, y = 29.2)

Q19. In a correlation study the following values are obtained:

Particulars	x	y
Mean	65	67
Standard Deviation	2.5	3.5
Co-efficient correlation	0.8	

Find the two regression equations that are associated with the above values. [Refer Similar, Q18]

(Ans: x = 0.57y + 26.72; y = 1.12 x – 5.8)

Q20. The following data about the sales and advertisement expenditure of a firm given below,

	Sales (₹ Crore)	Adv.Exp (₹ Crore)
Mean	40	6
S.D	10	1.5
Coefficient correlation		r = + 0.9

(i) What should be the advertisement expenditure if the firm proposed a sales targets of 50 crores of rupees?

(ii) Estimate the likely sales for a proposed advertisement expenditure of 12 crores of rupees. [Refer Similar, Q24]

(Ans: x = 0.135y + 0.6; y = 6x + 4; y = 304; x = 2.222)

Q21. Following data are given for marks in english (x) and marks in maths (y) at a certain examination.

	x	y
Mean marks	39.5	47.5
S.D of marks	10.8	16.8
Co-efficient correlation is	0.42	

Find the regression equation of y on x and estimate the marks in maths when marks in english is 50. [Refer Similar, Q24]

(Ans: y = 0.653 x – 21.70; y = 10.95)

Q22. The correlation Co-efficient between x and y is (r) = 0.6, σ_x = 1.5, σ_y = 2, $\overline{X}$ = 10 and $\overline{Y}$ = 20, find the two Regression Equations. [Refer Similar, Q17] Oct./Nov.-16, Q12(b) (OU)

(Ans: x = 0.45y + 1; y = 0.799x + 12.01)

INTERNAL ASSESSMENT/EXAM

I Multiple Choice

1. _________ is the study of mathematically measuring the average relationships, if it exists between two or more variables. [ ]

   (a) Correlation analysis (b) Regression analysis

   (c) Correlation coefficient (d) Correlation ratios

2. The variable which influences the values of other variable, [ ]

   (a) Dependent variable (b) Regression

   (c) Independent variable (d) Regression coefficient

3. The variable whose value is influenced or is to be predicted, [ ]

   (a) Dependent variable (b) Regression

   (c) Regression coefficient (d) Independent variable

4. If the regression curve is a straight line, then the regression is termed as _________ regression. [ ]

   (a) Simple (b) Multiple

   (c) Non-linear (d) Linear

5. Standard error of estimate is also known as _________. [ ]

   (a) Standard error (b) Standard error of regression

   (c) Standard error of prediction (d) None of the above

6. If the curve of the regression is not a straight line, then the regression is termed as _________ regression. [ ]

   (a) Simple (b) Multiple

   (c) Non-linear (d) Linear

7. _________ uses the principle of least squares to give the best fit line for estimating the value of one variable given the value of another variable. [ ]

   (a) Standard error of estimate (b) Regression coefficient

   (c) Regression equation (d) Lines of regression

8. _________ consists of minimizing the sum of the squares of the residuals or error of estimates. [ ]

   (a) Principle of least square (b) Lines of regression

   (c) Regression equation (d) Regression coefficient

9. If the regression lines are expressed in an algebra terms, then it is called as _________. [ ]

   (a) Regression (b) Regression equation

   (c) Regression coefficient (d) None of the above

10. For two variables X and Y, there are _________ lines of regression. [ ]

   (a) One (b) Three

   (c) Two (d) Four

II Fill in the Blanks

1. _________ studies nature of relationship between the variables.
2. Regression takes its name from studies made by _________.
3. _________ are independent of the change of the origin but not the change of scale.
4. Regression analysis involves two types of variables i.e., _________ and _________ variables.
5. The least square regression line always passes through _________.
6. Regression analysis is also used in _________.
7. The equation of _________ is commonly use to predict the value of Y for a given value of X.
8. __________ indicate the average values of one variable for a given or known values of other variable.
9. When the degree of correlation is high, then the _________ of regression will be close to each other.
10. There are two ways of forming regression equations, i.e., _________ and regression coefficient.

KEY

I. Multiple Choice

1. (b)
2. (c)
3. (a)
4. (d)
5. (b)
6. (c)
7. (d)
8. (a)
9. (b)
10. (c)

II. Fill in the Blanks

1. Regression
2. Sir Francis Galton
3. Regression coefficient
4. Dependent and independent
5. $(\bar{x}, \bar{y})$
6. Optimization
7. Regression line
8. Straight line
9. Two lines
10. Normal equation.

III Very Short Questions and Answers

Q1. What is Regression Analysis?

Answer :

According to Ya-Lun Chou, "regression analysis attempts to establish the 'nature of the relationship' between variables that is, to study the functional relationship between the variables and thereby provide a mechanism for prediction or forecasting".

Q2. What do you mean by Linear Regression?

Answer :

If the regression curve is a straight line, then the regression is termed as linear regression. The equation of such a curve is the equation of a straight line i.e., first degree equation in variables x and y.

Q3. Write a short note on lines of regression.

Answer :

In a bi-variate distribution, if the variables are related, then the points when plotted in the scatter diagram will lie near a straight line which is called the line of regression.

Q4. What do you mean by non-linear regression?

Answer :

If the curve of the regression is not a straight line, then the regression is termed as curved or non-linear regression. The regression equation will be a functional relation between variables x and y involving terms in x and y of degree more than one.

Q5. What is Simple Regression?

Answer :

The regression analysis confined to the study of only two variables at a time is termed as simple regression.

UNIT 2

Index Numbers

SYLLABUS

Introduction - Uses - Types - Problems in the Construction of Index Numbers - Methods of Constructing Index Numbers - Simple and Weighted Index Number (Laspeyre - Paasche, Marshall – Edgeworth) - Tests of Consistency of Index Number: Unit Test - Time Reversal Test - Factor Reversal Test - Circular Test - Base Shifting - Splicing and Deflating of Index Numbers.

LEARNING OBJECTIVES

- ✓ *Introduction, Characteristics, Uses, Limitations and Types of Index Numbers.*
- ✓ *Problems in the Construction of Index Numbers.*
- ✓ *Methods of Constructing Index Numbers (Laspeyre's, Paasche, Marshall-Edgeworth and Fishers).*
- ✓ *Tests of Consistency of Index Numbers.*
- ✓ *Base Shifting, Splicing and Deflating of Index Numbers.*

INTRODUCTION

Index numbers are numerical values or devices or series of numbers that measure the average changes in price or quantity over a period of time. Index numbers are expressed in percentages. They are calculated by dividing current value by the base value and then multiplied by 100. Index numbers do not have units.

Index numbers are helpful in measuring fluctuations in the economic conditions. They act as a barometer and measure the pressure of economic and business behaviour.

The various methods of constructing index numbers are: Weighted Price Indexes or Indices, Simple Price Indexes or Indices (Unweighted Price Indexes or Indices).

Laspeyre's index method was introduced by a famous statistician named "Laspeyre". In this method, prices of all items or commodities are weighed by the quantity, consumed both in the base and the current year.

In Paasche's index method, prices are weighted by the quantity consumed in the current year, rather than considering quantity consumed in base year.

In Marshall-Edgeworth's index method, both the current year as well as base year quantities are considered to calculate the index.

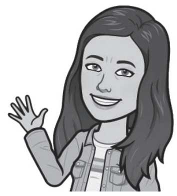
A statistician named "Fisher" introduced this method, which is a geometric mean of Laspeyre's and Paasche's methods. Fisher's ideal index takes into consideration both the base year and current year quantities.

The various tests of consistency of index numbers are as follows, Unit Test, Time Reversal Test, Factor Reversal Test, Circular Test.

Deflating refers to the process of making allowances for the impact of changing prices. An increase in price level leads to a decrease in purchasing power of money.

PART-A SHORT QUESTIONS AND ANSWERS

Q1. Define Index Numbers.



Answer : Model Paper-II, Q2

According to Maslow, "Index number is a numerical value characterizing the change in complex economic phenomena over a period of time or space".

According to Corxton and Cowden, "Index numbers are devices measuring differences in the magnitude of a group of related variables".

According to Horace Secrist, "Index numbers are series of numbers by which changes in the magnitude of a phenomenon are measured from time to time or place to place".

From the above definitions, we can define index number as "Index number is a specialized average designed to measure the change of related variable over a period of time".

Q2. Importance of Index Numbers (Model Paper-I, Q2 | May/June-18, Q2 (OU))

OR

Advantages of Index Numbers March/April-17, Q8 (OU)

OR

Explain the uses of Index Numbers.

Answer : March/April-14, Q7 (OU)

Following are the advantages, uses or importance of index numbers,

1. Helps in Measuring Economic Conditions

Index numbers are helpful in measuring fluctuations in the economic conditions. They act as a barometer and measure the pressure of economic and business behaviour. For example, the composite index numbers of foreign exchange reserves, industrial output and bank deposits can act as an economic barometer.

2. Policy Formulation

Price index reveals the fluctuations in prices over a period of time. Movement in prices directly effects the business as well as economic operations. Price index guides the business organizations and governmental bodies in formulating a new policy or modifying the existing policy in order to meet the issues that may arise due to price fluctuations.

3. Deflating Various Values

Index numbers can also be used for deflating i.e., they are useful to adjust original data for price changes. For example, various values such as national income of the population.

4. Inflationary and Deflationary Tendencies

Now-a-days index numbers are also being widely used for measuring the inflation and deflation. They are acting as indicators for inflationary and deflationary tendencies.

Q3. Types of Index Numbers

Answer : (Model Paper-III, Q3 | May/June-19, Q2 (OU) | Oct./Nov.-14, Q4 (OU))

Index numbers are classified into three types as follows,

1. Price Index Numbers

This method is used to compare the prices of various commodities of one year with that of the another year. It is further classified into two types,

(i) Wholesale Price Index Numbers

(ii) Retail Price Index Numbers.

2. Quantity Index Numbers

Quantity index numbers measure the variation in the volume of goods manufactured or consumed or distributed in quantitative terms. It measures the change in quantitative terms like, increase or decrease in volume of production, construction or employment.

3. Value Index Numbers

Value index numbers are used to measure the changes that takes place in the total value of the variable. The total value of the variable is obtained by multiplying price (p) with quantity (q).

$$\boxed{\text{Value} = \text{Price} \times \text{Quantity}}$$

Q4. Marshall EdgeWorth Method

Answer : May/June-19, Q2 (MGU)

In Marshall-Edgeworth's index method, both the current year as well as base year quantities are considered as to calculate the index. The formula for Marshall-Edgeworth's Index method is as follows,

$$I_p\,(ME) = \frac{\Sigma(q_0+q_1)P_1}{\Sigma(q_0+q_1)P_0}\times 100 = \left(\frac{\Sigma P_1 q_0 + \Sigma P_1 q_1}{\Sigma P_0 q_0 + \Sigma P_0 q_1}\right)\times 100$$

The following are the advantages of Marshall-Edgeworth's index method,

1. It is easy to understand and calculate.
2. It uses current and base year's quantities and prices.
3. It satisfies both the unit and time reversal tests of consistency.

Q5. What is "Cost of living Index"?

Answer : May/June-18, Q1(b) (KU)

Consumer Price Index numbers is also known as Cost of Living Index number is used to measure the purchasing power of a particular class of people in relation to the changes in retail prices. In other words, it studies how price variations effect the cost of living or purchasing power of a group of people.

While constructing cost of living index number, a particular section of society is selected like [Rich, middle, poor] and a study is conducted to know how price variations effect the consumption levels of that section. Based on such information Cost of Living Index Number (CLIN) is constructed.

Q6. Calculate index number by simple aggregative method.

	A	B	C	D
Price in 2005 (₹)	162	256	257	132
Price in 2007 (₹)	171	164	189	145

Answer : (Model Paper-I, Q3 | Oct./Nov.-12, Q8 (OU))

Computation of Price Index Number

Commodity	Price (in ₹)	
	2005 (p_0)	2007(p_1)
A	162	171
B	256	164
C	257	189
D	132	145
Total	$\Sigma p_0 = 807$	$\Sigma p_1 = 669$

I am Simple and Easy

Calculation of index number by using simple aggregate method,

$$P_{01} = \frac{\sum p_1}{\sum p_0}\times 100 = \frac{669}{807}\times 100 = 0.8289\times 100$$

$$= 82.90$$

Q7. The prices for period 2016 to 2018 are given below:

Year	Price
2016	25
2017	26
2018	27

Calculate simple price index taking 2016 as base year.

Answer :

Simple price index, $P_{0n} = \frac{P_n}{P_0} \times 100$

$P_{2016,\,2016} = \frac{25}{25} \times 100 = 100\%$

$P_{2016,\,2017} = \frac{26}{25} \times 100 = 104\%$

$P_{2016,\,2018} = \frac{27}{25} \times 100 = 108\%$

Q8. From the following data calculate a price index based on price relatives method using Arithmetic Mean.

Commodity	A	B	C	D	E	F
Price 2015 (₹)	45	60	20	50	85	120
Price 2016 (₹)	55	70	30	75	90	130


If I Don't Come,
My Method Will Come

Answer : (Model Paper-II, Q3 | May/June-18, Q3 (OU))

Calculation of Price Index Based on Price Relatives Method by Using Arithmetic Mean

Commodities	Prices (₹)		Price Relatives $\left[\frac{P_1}{P_0}\right]100$
	2015 (P_0)	2016 (P_1)	
A	45	55	122.22
B	60	70	116.67
C	20	30	150
D	50	75	150
E	85	90	105.88
F	120	130	108.33
N = 6			$\Sigma\left[\frac{P_1}{P_0}\right]100 = 753.1$

$$P_{01} = \frac{\Sigma\left[\frac{P_1}{P_0}\right]100}{N}$$

$$= \frac{753.1}{6}$$

$$= 125.517$$

Q9. Calculate Index number by Average Price Relative Method by using Arithmetic Mean.

Commodity	P	Q	R	S	T
Price 2017	2	6	10	5	12
Price 2018	4	8	15	5	8

Answer : May/June-19, Q3 (OU)

Calculation of Index Number by Average Price Relative Method Using Arithmetic Mean

Commodity	Prices		Price Relatives
	2017 (p_0)	2018 (p_1)	$\left[\frac{p_1}{p_0}\right]100$
P	2	4	200
Q	6	8	133.3
R	10	15	150
S	5	5	100
T	12	8	66.7
N = 5			$\Sigma\left[\frac{p_1}{p_0}\right]100 = 650$

$$P_{01} = \frac{\Sigma\left[\frac{P_1}{P_0}\right]100}{N} = \frac{650}{5} = 130$$

Q10. Compute price index by weighted average of price relatives method using arithmetic mean.

Commodities	A	B	C	D	E
Price in base year	10	6	14	22	18
Quantity in base year	160	180	120	40	80
Price in current year	16	8	14	28	24

Answer : March/April-15, Q8 (OU)

Calculation of Price Index by Weighted Average of Price Relatives Method using Arithmetic Mean

Commodities	p_0	q_0	p_1	p_0q_0 V	$\frac{p_1}{p_0}\times 100$ P	PV
A	10	160	16	1,600	160	2,56,000
B	6	180	8	1,080	133.33	1,43,996.4
C	14	120	14	1,680	100	1,68,000
D	22	40	28	880	127.27	1,11,997.6
E	18	80	24	1,440	133.33	1,91,995.2
				ΣV = 6,680		PV = 8,71,989.2

$$p_{01} = \frac{\Sigma PV}{\Sigma V} = \frac{8,71,989.2}{6680}$$

$$= 130.537$$

∴ It means that, there is a increase in prices over base level by 130.537%.

PART-B ESSAY QUESTIONS AND ANSWERS

2.1 INDEX NUMBERS – INTRODUCTION, CHARACTERISTICS, USES, LIMITATIONS AND TYPES

Q11. Define Index Number. What are its features and uses?

(Model Paper-II, Q10(a) | May/June-19, Q10(a) (OU))

OR

What is Index Number? Write importance of index numbers.

(Refer Only Topics: Index Numbers, Uses/ Importance of Index Numbers)

Answer : May/June-18, Q3(a) (KU)

Index Numbers

According to Maslow, "Index number is a numerical value characterizing the change in complex economic phenomena over a period of time or space".

According to Corxton and Cowden, "Index numbers are devices measuring differences in the magnitude of a group of related variables".

According to Horace Secris, "Index numbers are series of numbers by which changes in the magnitude of a phenomenon are measured from time to time or place to place".

Index numbers are numerical values or devices or series of numbers that measure the average changes in price or quantity over a period of time. Index numbers are expressed in percentages and also they do not have units.

Features/Characteristics of Index Numbers

The features or characteristics of index numbers are as follows,

1. Measure the Change in Percentages

Index numbers measure the change in price or quantity in terms of percentages such as, 10%, 20%, 15%, 25% and so on. Increase or decrease in value is represented by one single figure. Like 10% increase in sales from that previous year to current year, 30% decrease in profits when compared to that of the last year and so on.

2. Specialized Averages

A single figure known as "Average" is used for representing the characteristics of the complete set of data. Average acts as a basis for comparing different data sets with each other, when they have common unit of measurement of observations. When data sets do not have common unit of measurement, specialized averages of index numbers are used for the comparison.

3. The Measured Changes cannot be Observed Directly

Index numbers do not measure the changes directly but it studies the relative changes or variations in factors resulting to changes-like for measuring changes in Export-Imports related factors such as available raw materials, technology, competitors etc.

4. Measures Changes in Relation to Time or Place

Index numbers measure change by comparing the values at different time periods or at different places like standard of living at one place is being compared with standard of living of the other place. Sales or revenue of current year is compared with that of the previous years sales or revenue.

Uses/Importance of Index Numbers

For answer refer Unit-II, Page No. 30, Q.No. 12, Topic: Uses/Importance of Index Numbers.

Q12. What is the importance and limitations of index numbers? Explain.

Answer : (Model Paper-I, Q10(a) | May/June-19, Q7(b) (MGU))

Uses/Importance of Index Numbers

The uses/importance of index numbers are as follows,

1. Helps in Measuring Economic Conditions

Index numbers are helpful in measuring fluctuations in the economic conditions. They act as a barometer and measure the pressure of economic and business behaviour. For instance the composite index numbers of foreign exchange reserves, industrial output and bank deposits can act as an economic barometer.

2. Measuring Purchasing Power

Purchasing power of a group or class can be easily measured with the help of the index numbers. As purchasing power is related with a group of people, price index is required for providing overall view of the purchasing power of the group. By considering earnings and expenses on purchases of a particular group. For example, index numbers can measure the fluctuations (increase or decrease) in purchasing power of that group, over a period of time.

3. Reveal Trend and Tendencies

Index numbers measure the average changes in phenomenon, by taking into account the current year and basic year values. The price index gives the fluctuations in specific years, through which trends of the phenomenon can be easily represented in graph. Even conclusions can also be framed by analyzing these trends.

4. Policy Formulation

Price index reveals the fluctuations in prices over a period of time. Movement in prices directly effects the business as well as economic operations. Price index guides the business organizations and governmental bodies in formulating a new policy or modifying the existing policy in order to meet the issues that may arise due to price fluctuations.

5. Deflating Various Values

Index numbers can also be used for deflating various values such as national income of the overall population.

6. Inflationary and Deflationary Tendencies

Now-a-days index numbers are also being widely used for measuring the inflation and deflation. They are acting as indicators for inflationary and deflationary tendencies.

Limitations of Index Numbers

The following are the various limitations of index numbers,

1. Difficulty in Selection of Data

It is difficult to include each and every item in the construction of index as such construction takes into consideration of only the selected sample data.

2. Random Sampling

In the construction of index numbers, random sampling is hardly used as selecting a sample from huge population as random sampling procedure is not practical. Inspite of selecting the samples carefully, some errors might take place in the construction of indices. Therefore, efforts must be made to reduce such errors.

3. Quality of Product

Under index numbers the quality of products should remain same during a period of time. Dissimilarity in quality of the products means dissimilarity in prices of products which makes the comparison during a period of time less authentic in nature.

4. Methods of Construction

Index numbers might by constructed by using different methods which gives different results. Further, it requires a selection of an appropriate formula, otherwise, it would result in problems of comparison of results. Due care should be taken while selecting a formula and same formula should be used over a period of time.

5. Misuse

It might be misused by some dishonest capitalists who can show less profits in the company. Current year profits are compared with the record year profits to show less profits in the company. Likewise, fraudulent trade compares the current year prices with the year where prices were very high to show current year prices as very high prices.

6. Inaccurate Information

Non- availability of accurate and sufficient information is one of the limitation in the construction of index numbers.

7. Others

(i) Data needs to be collected from geographically scattered locations.

(ii) The chances of fake results are high.

(iii) The price of one commodity might differ from the prices of another commodities due to different prices fixed by the wholesalers or retailers.

Q13. Explain the various types of Index Numbers.

Answer :

Index numbers are classified into three types as follows,

1. Price index numbers
2. Quantity index numbers and
3. Value index numbers.

1. Price Index Numbers

Price index method is used to compare the prices of various commodities of one year with that of the another year. It is further classified into two types. They are as follows,

(i) Wholesale Price Index Numbers

Wholesale price indices are used to study the changes that takes place in the general price level of a country.

(ii) Retail Price Index Numbers

It is used to study the changes that takes place in retail prices of various commodities like fruits, vegetables, rice, wheat etc.

2. Quantity Index Numbers

Quantity index numbers measure the variation in the volume of goods manufactured or consumed or distributed in quantitative terms. It measures the change in quantitative terms like, increase or decrease in volume of production, construction or employment.

3. Value Index Numbers

Value index number are used to measure the changes that takes place in the total value of the variable. The total value is obtained by multiplying price (p) with quantity (q).

Value = Price × Quantity

2.2 PROBLEMS IN THE CONSTRUCTION OF INDEX NUMBERS

Q14. Discuss the problems in the construction of Index Numbers.

Answer :

While constructing index numbers the following problems must be taken into consideration,

1. Purpose/Object of Index

While constructing index number the purpose/ object should be clearly decided.

For example, if consumer's cost of living index number is used to measure standard of living of poor families then in such case utmost care should be taken that it should not include any other data of middle class and rich class in it. Otherwise it would result in confusion and wastage of time and accurate results would not be obtained.

2. Selection of a Base Period

A base period is selected to compare the two periods. It might be either a year a month or a day. Index for base period should be always taken as 100. While selecting a base period the following points should be taken into account.

(i) A base period selected should be free from abnormalities such as earthquakes, wars, famines, depression booms and so on. But some times selecting a base period which is free from all abnormalities becomes a difficult task.

(ii) A selected base year should not be too far from the current year.

(iii) While constructing price index number a decision has to be taken whether to proceed with fixed base or chain base index. Under fixed base method a fixed year is selected for the entire series. Whereas in chain base method, price are compared with their preceding year instead of the fixed year.

However, chain base method gives better results when compared to the fixed base method.

3. Number of Commodities to be Included

In construction of price index number only those commodities are taken into consideration which best represent the tastes, preferences of the consumers and habits of the people for whom index is constructed.

For example, if price index number is used to measure the monthly budget of a particular family. Then in such a case it should include only items like,

Clothing, fuel and light. Apart from these no other items should be included. The selection of number of commodities to be included is decided based upon the purpose of the index.

4. Price Quotations

After the selection of commodities the next problem which arises is collection of the accurate price quotations for such commodities. As prices of commodities differ from place to place and from shop to shop within the market it becomes impossible to get price quotations from all such places where commodity is manufactured. So respective persons and places should be selected. The data should be obtained from reliable sources like journal, magazine newspapers and government organizations. Therefore, in order to ensure uniformity in prices, two methods of quoting prices are followed. They are,

(i) Money prices and

(ii) Quantity prices.

Price quotations are available in two forms i.e., wholesale and retail. A decision regarding whether to select wholesale prices or retail prices are made based upon the objective, purpose of index number.

5. Choice of an Average (Two Contradictory Statements are Given)

Before constructing index numbers, selecting an average is mandatory. In practice, median, mode and mean are not used for constructing the price index. Among all averages, geometric mean is considered as the most suitable one for construction of index numbers because of the following reasons,

(i) In the construction of index numbers focus is laid on finding out the ratios of change.

(ii) Results obtained by using geometric mean are reversible. Thereby it facilitates base shifting.

6. Selection of an Appropriate Formula

Selection of an appropriate formula is made on the basis of the purpose of the index and available data. According to Prof. Irving Fisher, an index is considered as appropriate, when it is both time reversal test and factor reversal test. However, in practice, there is no particular formula which can be considered as appropriate in situations.

7. Selection of Appropriate Weights

It is important to select appropriate weights for the items to measure the relative importance of different items in the construction of index numbers. Indices can be broadly categorized into two types. They are as follows,

(i) Weighted indices and

(ii) Unweighted indices.

In case of weighted indices, particular weights are assigned to the items. But in case of unweighted indices no specific weights are assigned to the items.

Weights can be assigned to the items by two methods. They are,

(a) Implicit method and

(b) Explicit method.

Ultimately, the decision needs to be taken regarding the types of weights like whether it should select fixed or fluctuating weights. Selection of appropriate weights is very important and also a very difficult task.

2.3 METHODS OF CONSTRUCTING INDEX NUMBERS - SIMPLE AND WEIGHTED INDEX NUMBER

Q15. What are the various methods of constructing index numbers? Explain.

Answer :

The various methods of constructing index numbers are shown in the following figure,

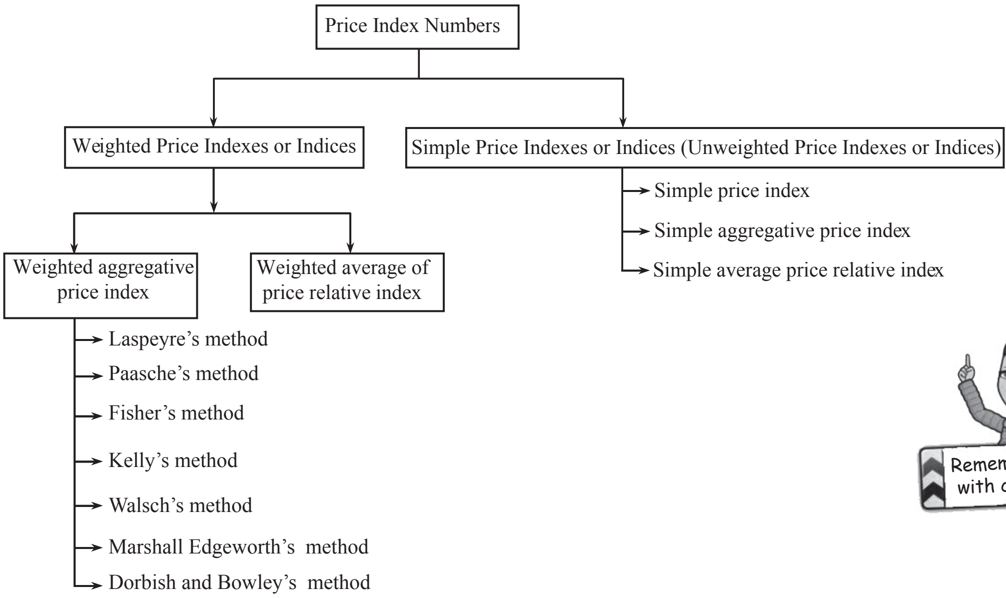


Figure: Types and Methods of Price Index Numbers

1. Weighted Price Indexes or Indices

At the time of constructing the weighted price indexes or indices, the rational weights are allocated in an explicit manner. These rational weights show the relative significance of items or commodities which are related with the computation of an index. Quantity weights and value weights are used in this weighted indexes or indices. Weighted price indexes or indices are further divided into two types. They are,

(a) Weighted Aggregate Price Index

In a weighted aggregate price index, certain weight is assigned to each and every commodity or item of group in accordance with its significance. This helps in gathering more information and improving accuracy of the estimates. The following methods are used in weighted aggregate price index,

(i) Laspeyre's method

(ii) Paasche's method

(iii) Fisher's ideal method

(iv) Kelly's method

(v) Walsch's method

(vi) Marshall Edgeworth's method

(vii) Dorbish and Bowley's method.

(b) Weighted Average of Price Relative Index

In weighted average of price relative index, value of each commodity or item related with the calculation of composite index is ascertained by multiplying the price of each item with its quantity consumed. Quantity consumed is considered for computing the weighted average of price relative. The formula for weighted average of price relative index is as follows,

$$P_{01} = \frac{\Sigma((p_1 \div p_0)\times 100)(p_0 q_0)}{\Sigma p_0 q_0}$$

$$= \frac{\Sigma PV}{\Sigma V}$$

$$= \frac{\Sigma p_1 q_0}{\Sigma p_0 q_0} \times 100$$

Where,

$V\,(= p_0 q_0)$ = Base prices and quantities determining values.

$$P\left(= \frac{p_1}{p_0}\times 100\right) = \text{Price relative}$$

This formula is equivalent to the formula of Laspeyre's index formula,

If 'V' is taken as $p_0\,q_1$, then the formula would,

$$P_{01} = \frac{\Sigma p_1 q_1}{\Sigma p_0 q_1} \times 100$$

Then it would be equal to Paasche's index method.

2. Simple/Unweighted Price Indexes or Indices

The simple or unweighted indexes or indices include the following methods,

(a) Simple/Single Price Index

Single price index is computed by dividing the current year price of the commodity with its base year price. It is a percentage ratio which represents the comparison of a particular commodity price. The general formula used for single price index is as follows,

$$\text{Single price index in period '}n\text{'} = \frac{p_n}{p_0} \times 100$$

Where,

p_n = Price of the commodity in the n^{th} year

P_0 = Price of the commodity in the base year.

(b) Simple Aggregate Price Index

In aggregate price index, the sum of current year prices of various commodities is divided with the sum of base year prices of that various commodities. The formula is given as follows,

$$\text{Aggregate price index, } P_{01} = \frac{\Sigma p_1}{\Sigma p_0} \times 100$$

Where,

p_1 = Unit price of a current year prices of all commodities

p_0 = Sum of base year prices of all commodities.

(c) Simple Average Price Relative Index

This method is an improvement over the aggregate price method. The formula for this method is,

$$P_{01} = \sum\left(\frac{P_1}{P_0} \times 100\right)$$

Where,

n = Number of commodities included in the computation of the index.

2.3.1 Laspeyre's Index Method

Q16. What is Laspeyre's Index Method? What are its advantages and disadvantages?

Answer :

Laspeyre's Index Method

Laspeyre's index method was introduced by a famous statistician named "Laspeyre". In this method, prices of all items or commodities are weighted by the quantity, consumed both in the base and the current year. Index of the different periods can be directly compared with each other in this method, as the index number relies upon the same base price and quantity. The formula for Laspeyre's price index method is as follows,

$$P_{01} = \frac{\Sigma p_1 q_0}{\Sigma p_0 q_0} \times 100$$

Where,

p_1 = Current year prices

p_0 = Base year prices

q_0 = Quantities consumed in base year.

Advantages of Laspeyre's Index Method

The following are the advantages of Laspeyre's index method,

1. It facilitates direct comparison of index of various periods with other periods.
2. It is not needed to maintain, a record of the consumed quantities in each period, as this method takes into consideration only single quantity measure on the basis of the base period.

Disadvantages of Laspeyre's Index Method

The disadvantages of Laspeyre's index method are as follows,

1. This method is upward biased and thus overestimate price levels and inflation.
2. Only base year quantities are considered for calculating price index.

2.3.2 Paasche's Index Method

Q17. What is Paasche's Index Method? What are its advantages and disadvantages?

Answer :

Paasche's Index Method

In Paasche's index method, prices are weighted by the quantity consumed in the current year, rather than considering quantity consumed in base year. The formula for Paasche's index method is,

$$P_{01} = \frac{\Sigma p_1 q_1}{\Sigma p_0 q_1} \times 100$$

Where,

p_1 = Current year prices

p_0 = Base year prices

q_1 = Quantities consumed in current year.

Advantages of Paasche's Index Method

The following are some of the advantages of Paasche's index method,

1. It integrates effects of changes in price and quantity consumed during the current year.
2. It gives a more effective estimation of changes of commodities or items when compared to the Laspeyre's method.

Disadvantages of Paasche's Index Method

The disadvantages of Paasche's index method are,

1. This method is costly and time consuming in nature as it needs to maintain record of quantities consumed in each year.
2. In every year, recomputation of previous years's index number is needed in order to show the effect of the new quantity weights.

2.3.3 Marshall-Edgeworth's Index Method

Q18. What is Marshall-Edgeworth's Index Method? What are its advantages and disadvantages?

Answer :

Marshall-Edgeworth's Index Method

In Marshall-Edgeworth's index method, both the current year as well as base year quantities are considered as to calculate the index. The formula for Marshall-Edgeworth's Index method is as follows,

$$I_P\,(ME) = \frac{\Sigma(q_0 + q_1)P_1}{\Sigma(q_0 + q_1)P_0} \times 100$$

$$= \left(\frac{\Sigma P_1 q_0 + \Sigma P_1 q_1}{\Sigma P_0 q_0 + \Sigma P_0 q_1}\right) \times 100$$

Advantages of Marshall-Edgeworth's Index Method

The following are the advantages of Marshall-Edgeworth's index method,

1. It is easy to understand and calculate.
2. It uses current and base year's quantities and prices.
3. It satisfies both the unit and time reversal tests of consistency

Disadvantages of Marshall-Edgeworth's Index Method

The following are the disadvantages of Marshall-Edgeworth's index method,

1. It needs current weights while constructing index numbers.
2. It is not satisfactory for circular and factor reversal test of consistency.

2.3.4 Fisher's Ideal Index Method

Q19. What is Fisher's Ideal Index Method? What are its advantages and disadvantages?

Answer :

Fisher's Ideal Index Method

A statistician named "Fisher" introduced this method, which is a geometric mean of Laspeyre's and Paasche's methods. The formula used for this method is,

$$P_{01} = \sqrt{\frac{\Sigma p_1 q_0}{\Sigma p_0 q_0} \times \frac{\Sigma p_1 q_1}{\Sigma p_0 q_1}} \times 100$$

Advantages of Fisher's Ideal Index Method

The following are some of the advantages of Fisher's ideal index method,

1. It takes into consideration both the base year and current year quantities.
2. It is based on geometric mean, which facilitates calculation of best index number.
3. It satisfies both Time reversal test and Factor reversal test.

Disadvantages of Fisher's Ideal Index Method

The disadvantages of Fisher's ideal index method are as follows,

1. It requires complicated computation which is lengthy in nature.
2. It is not suitable for general use.

2.3.5 Quantity Index Numbers

Q20. Write about quantity index numbers. What are the various methods used for constructing quantity index numbers?

Answer :

Quantity Index Numbers

Quantity Index numbers measures the variation in the volume of goods manufactured or consumed or distributed in quantitative terms. It measures the change in quantitative terms like, increase or decrease in volume of production construction or employment.

While constructing quantity index numbers, current year production or sales data is compared with the base year data. In order to measure the changes in quantities, the values or prices are taken as weights. The formulae for quantity index numbers can be obtained from the formulae used in index numbers by taking (p) in place of (q) and taking (q) in place of (p).

Methods for Constructing Quantity Index Numbers

The following are the three methods which are used for constructing quantity index numbers,

(i) Laspeyre's Quantity Index Method

$$Q_{01} = \frac{\sum q_1 p_0}{\sum q_0 p_0} \times 100$$

Where,

q_1 = Quantity consumed in the current year.

q_0 = Quantity consumed in the base year.

p_0 = Base year prices.

(ii) Paasche's Quantity Index Method

$$Q_{01} = \frac{\sum q_1 p_1}{\sum q_0 p_1} \times 100$$

Where,

q_1 = Quantity consumed in the current year.

q_0 = Quantity consumed in the base year.

p_1 = Current year prices.

(iii) Fisher Quantity Index Method

$$Q_{01} = \sqrt{\frac{\sum q_1 p_0}{\sum q_0 p_0} \times \frac{\sum q_1 p_1}{\sum q_0 p_1}} \times 100$$

PROBLEMS ON INDEX NUMBERS

Q21. From the following data, calculate price index number by,

(i) Laspeyre's method

(ii) Paasche's method

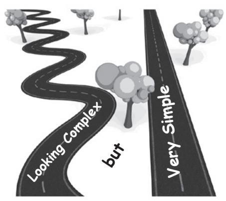
(iii) Fisher's ideal method

(iv) Kelly's method

(v) Walsch's method

(vi) Marshall Edgeworth's method

(vii) Dorbish and Bowleys method.



Commodity	2005		2009	
	Price ₹	Qty	Price ₹	Qty
A	20	8	40	6
B	50	10	60	5
C	40	15	50	15
D	20	20	20	25

Solution :

Commodity	p_0	q_0	p_1	q_1	p_1q_0	p_0q_0	p_1q_1	p_0q_1	q_0q_1	$\sqrt{q_0q_1}$	$p_1\cdot\sqrt{q_0q_1}$	$p_0\cdot\sqrt{q_0q_1}$	$q=\frac{q_0+q_1}{2}$	p_0q	p_1q
A	20	8	40	6	320	160	240	120	48	6.93	277.2	138.6	7	140	280
B	50	10	60	5	600	500	300	250	50	7.07	424.2	353.5	7.5	375	450
C	40	15	50	15	750	600	750	600	225	15	750	600	15	600	750
D	20	20	20	25	400	400	500	500	500	22.36	447.2	447.2	22.5	450	450
Total					Σp_1q_0 = 2070	Σp_0q_0 = 1660	Σp_1q_1 = 1790	Σp_0q_1 = 1470			$\Sigma p_1\sqrt{q_0q_1}$ = 1898.6	$\Sigma p_0\sqrt{q_0q_1}$ = 1539.3		Σp_0q = 1565	Σp_1q = 1930

(i) Laspeyre's Index Method

$$P_{01} = \frac{\Sigma p_1q_0}{\Sigma p_0q_0} \times 100$$

$$= \frac{2070}{1660} \times 100$$

$$= 124.7$$

(ii) Paasche's Index Method

$$P_{01} = \frac{\Sigma p_1q_1}{\Sigma p_0q_1} \times 100$$

$$= \frac{1790}{1470} \times 100$$

$$= 121.8$$

(iii) Fisher's Ideal Index Method

$$P_{01} = \sqrt{\frac{\Sigma p_1q_0}{\Sigma p_0q_0} \times \frac{\Sigma p_1q_1}{\Sigma p_0q_1}} \times 100$$

$$= \sqrt{\frac{2070}{1660} \times \frac{1790}{1470}} \times 100$$

$$= \sqrt{1.247 \times 1.218} \times 100$$

$$= 123.24$$

(iv) Kelly's Method

$$P_{01} = \frac{\Sigma p_1q}{\Sigma p_0q} \times 100$$

$$= \frac{1930}{1565} \times 100$$

$$= 123.32$$

(v) Walsch's Method

$$P_{01} = \frac{\Sigma p_1\sqrt{q_0q_1}}{\Sigma p_0\sqrt{q_0q_1}} \times 100$$

$$= \frac{1898.6}{1539.3} \times 100$$

$$= 123.34$$

(vi) Marshall Edgeworths Method

$$P_{01} = \left[\frac{\Sigma p_1q_0 + \Sigma p_1q_1}{\Sigma p_0q_0 + \Sigma p_0q_1}\right] \times 100$$

$$= \left[\frac{2070 + 1790}{1660 + 1470}\right] \times 100$$

$$= \left[\frac{3860}{3130}\right] \times 100$$

$$= 123.32$$

(vii) Dorbish and Bowley's Method

$$P_{01} = \frac{1}{2}\left[\frac{\Sigma p_1q_0}{\Sigma p_0q_0} + \frac{\Sigma p_1q_1}{\Sigma p_0q_1}\right] \times 100$$

$$= \frac{1}{2}\left[\frac{2070}{1660} + \frac{1790}{1470}\right] \times 100$$

$$= \frac{1}{2}[1.247 + 1.218] \times 100$$

$$= \frac{1}{2}[2.465] \times 100$$

$$= 123.25$$

Q22. Compute Price Index Number by using:

(i) Paasches and

(ii) Marshal and Edgeworth methods.

Commodity	Base Year		Current Year	
	Price	Quantity	Price	Quantity
P	5	100	6	150
Q	4	80	5	100
R	2	60	5	72
S	12	30	9	33

Solution : (Model Paper-I, Q10(b) | May/June-19, Q10(b) (OU))

Commodity	Base Year		Current Year		p_1q_0	p_0q_0	p_0q_1	p_1q_1
	Price (p_0)	Quantity (q_0)	Price (p_1)	Quantity (q_1)				
P	5	100	6	150	600	500	750	900
Q	4	80	5	100	400	320	400	500
R	2	60	5	72	300	120	144	360
S	12	30	9	33	270	360	396	297
					1,570	**1300**	**1,690**	**2,057**

(i) Paasche's Index Method

$$P_{01} = \frac{\Sigma p_1 q_1}{\Sigma p_0 q_1} \times 100$$

$$= \frac{2,057}{1,690} \times 100$$

$$= 1.217 \times 100$$

$$= 121.7$$

(ii) Marshal-Edgeworth's Index Method

$$P_{01} = \frac{\Sigma p_1 q_0 + \Sigma p_1 q_1}{\Sigma p_0 q_0 + \Sigma p_0 q_1} \times 100$$

$$= \frac{1,570 + 2,057}{1,300 + 1,690} \times 100$$

$$= \frac{3,627}{2,990} \times 100$$

$$= 1.213 \times 100$$

$$= 121.3$$

Q23. From the following data, calculate price index number by,

(i) Laspeyre's

(ii) Paasche's

(iii) Fisher's ideal method.

Commodity	2005		2009	
	Price ₹	Qty	Price ₹	Qty
A	20	8	40	6
B	50	10	60	5
C	40	15	50	15
D	20	20	20	25

Solution : (March/April-12, Q13(b) (OU) | March/April-11, Q13(b) (OU))

Commodity	p_0	q_0	p_1	q_1	p_1q_0	p_0q_0	p_1q_1	p_0q_1
A	20	8	40	6	320	160	240	120
B	50	10	60	5	600	500	300	250
C	40	15	50	15	750	600	750	600
D	20	20	20	25	400	400	500	500
Total					$\Sigma p_1q_0 = 2070$	$\Sigma p_0q_0 = 1660$	$\Sigma p_1q_1 = 1790$	$\Sigma p_0q_1 = 1470$

(i) Laspeyre's Index Method

$$P_{01} = \frac{\Sigma p_1 q_0}{\Sigma p_0 q_0} \times 100$$

$$= \frac{2070}{1660} \times 100$$

$$= 124.70$$

(ii) Paasche's Index Method

$$P_{01} = \frac{\Sigma p_1 q_1}{\Sigma p_0 q_1} \times 100$$

$$= \frac{1790}{1470} \times 100$$

$$= 121.77$$

(iii) Fisher's Ideal Index Method

$$P_{01} = \sqrt{\frac{\Sigma p_1 q_0}{\Sigma p_0 q_0} \times \frac{\Sigma p_1 q_1}{\Sigma p_0 q_1}} \times 100$$

$$= \sqrt{\frac{2070}{1660} \times \frac{1790}{1470}} \times 100$$

$$= \sqrt{1.247 \times 1.218} \times 100$$

$$= \sqrt{1.5188} \times 100$$

$$= 1.2324 \times 100$$

$$= 123.24$$

Q24. Calculate Fisher's idle index from the following data.

Goods	A	B	C	D	E
P_0(₹)	18	14	16	10	12
Total Cost	1000	600	480	840	720
P_1(₹)	18	16	14	18	20
Total Cost	2,400	960	1050	900	800

Solution : (Model Paper-III, Q10(a) | May/June-19, Q7(a) (MGU))

In the given problem, value of price and total cost is given. So, first we need to find out the quantity of the base year (q_0) and current year (q_1) by using the following formula,

$$\text{Quantity} = \frac{\text{Total Cost}}{\text{Price}}$$

The values of 'q_0' and 'q_1' are calculated as,

Goods	q_0	q_1
A	$\frac{1000}{18} = 55.56$	$\frac{2400}{18} = 133.33$
B	$\frac{600}{14} = 42.86$	$\frac{960}{16} = 60$
C	$\frac{480}{16} = 30$	$\frac{1050}{14} = 75$
D	$\frac{840}{10} = 84$	$\frac{900}{18} = 50$
E	$\frac{720}{12} = 60$	$\frac{800}{20} = 40$

Calculation of Fisher's Ideal Index

Goods	p_0	q_0	p_1	q_1	p_1q_0	p_0q_0	p_1q_1	p_0q_1
A	18	55.56	18	133.33	1000.08	1000.08	2399.94	2399.94
B	14	42.86	16	60	685.76	600.04	960	840
C	16	30	14	75	420	480	1050	1200
D	10	84	18	50	1512	840	900	500
E	12	60	20	40	1200	720	800	480
					Σp_1q_0 = **4817.84**	Σp_0q_0 = **3640.12**	Σp_1q_1 = **6109.94**	Σp_0q_1 = **5419.94**

Fisher's ideal price index,

$$p_{01} = \sqrt{\frac{\Sigma p_1q_0}{\Sigma p_0q_0} \times \frac{\Sigma p_1q_1}{\Sigma p_0q_1}} \times 100$$

$$= \sqrt{\frac{4,817.84}{3,640.12} \times \frac{6,109.94}{5,419.94}} \times 100$$

$$= \sqrt{1.3235 \times 1.1273} \times 100$$

$$= \sqrt{1.492} \times 100$$

$$= 1.2215 \times 100$$

$$= 122.15$$

Solve Me with Concentration

Q25. From the following data calculate Price Index Number by using (i) Paasche's Method and (ii) Marshal Edgeworth Method.

Item	Base Year		Current Year	
	Price (₹)	Expenditure (₹)	Price (₹)	Expenditure (₹)
P	6	300	10	560
Q	2	200	2	240
R	4	240	6	360
S	10	300	12	288
T	3	120	8	240

Solution : May/June-18, Q10(b) (OU)

Note: As price and expenditure are given for base year and current year, divide expenditure of each commodity with their respective price to obtain quantity of base year (i.e., q_0) and quantity of current year (i.e., q_1).

Item	Base year Price (p_0)	Base year Expenditure ($p_0 q_0$)	Current year Price (p_1)	Current year Expenditure ($p_1 q_1$)	$q_0 = \frac{p_0 q_0}{p_0}$	$q_1 = \frac{p_1 q_1}{p_1}$	$p_1 q_0$	$p_0 q_1$
P	6	300	10	560	50	56	500	336
Q	2	200	2	240	100	120	200	240
R	4	240	6	360	60	60	360	240
S	10	300	12	288	30	24	360	240
T	3	120	8	240	40	30	320	90
		1,160		**1,688**			**1,740**	**1,146**

(i) Paasche's Method

$$P_{01} = \frac{\Sigma p_1 q_1}{\Sigma p_0 q_1} \times 100$$

$$= \frac{1,688}{1,146} \times 100$$

$$= 147.29$$

(ii) Marshal Edgeworth Method

$$P_{01} = \frac{\Sigma p_1 q_0 + \Sigma p_1 q_1}{\Sigma p_0 q_0 + \Sigma p_0 q_1} \times 100$$

$$= \frac{1,740 + 1,688}{1,160 + 1,146} \times 100$$

$$= \frac{3,428}{2,306} \times 100$$

$$= 148.66$$

Q26. Compute quantity index numbers for the year 2010 with 2005 as base year using,

(i) Laspeyer's method

(ii) Paasche's method

(iii) Fisher's method.

Commodity	Quantity	(units)	Price (₹)	
	2005	2010	2005	2010
A	100	150	5	6
B	80	100	4	5
C	60	72	2.5	5
D	30	33	12	9

Solution :

Calculation of Quantity Index Numbers

Commodity	p_0	q_0	p_1	q_1	$q_0 p_1$	$q_0 p_0$	$q_1 p_0$	$q_1 p_1$
A	5	100	6	150	600	500	750	900
B	4	80	5	100	400	320	400	500
C	2.5	60	5	72	300	150	180	360
D	12	30	9	33	270	360	396	297
					$\Sigma q_0 p_1 = 1570$	$\Sigma q_0 p_0 = 1330$	$\Sigma q_1 p_0 = 1726$	$\Sigma q_1 p_1 = 2057$

(i) Laspeyre's Quantity Index

$$Q_{01} = \frac{\Sigma q_1 p_0}{\Sigma q_0 p_0} \times 100 = \frac{1726}{1330} \times 100$$

$$= 129.77$$

(ii) Paasche's Quantity Index

$$Q_{01} = \frac{\Sigma q_1 p_1}{q_0 p_1} \times 100 = \frac{2057}{1570} \times 100$$

$$= 131.02$$

(iii) Fisher's Quantity Index

$$Q_{01} = \sqrt{\frac{\Sigma q_1 p_0}{\Sigma q_0 p_0} \times \frac{\Sigma q_1 p_1}{\Sigma q_0 p_1}} \times 100$$

$$= \sqrt{\frac{1726}{1330} \times \frac{2057}{1570}} \times 100 = \sqrt{1.2977 \times 1.3102} \times 100$$

$$= \sqrt{1.7002} \times 100$$

$$= 1.3039 \times 100$$

$$= 130.39$$

2.4 TESTS OF CONSISTENCY OF INDEX NUMBER – UNIT TEST, TIME REVERSAL TEST, FACTOR REVERSAL TEST AND CIRCULAR TEST

Q27. Explain various tests of consistency of index numbers.

OR

Discuss the following,

(i) Unit test

(ii) Time reversal test.

(iii) Factor reversal test

(iv) Circular test.

Answer :

Price index numbers have various weighted and unweighted index methods. So, it is difficult to select a suitable method while constructing an index number. In order to overcome this issue, statisticians have suggested few tests for testing the adequacy or consistency of an index number. These tests include,

(i) Unit test

(ii) Time reversal test.

(iii) Factor reversal test

(iv) Circular test.

(i) Unit Test

According to unit test, the formula of index number should be independent of the units under which prices and quantities are quoted. All formulae satisfy this test except simple aggregative test.

(ii) Time Reversal Test

Time reversal test is basically used at checking whether the selected method would work for both forward and backward or not. According to this test, the formula should give exact ratio when compared with one point with the another i.e., for example,

$$P_{01} = \frac{1}{P_{01}} \text{ or } P_{01} \times P_{10} = 1$$

$$Q_{01} \times Q_{10} = 1$$

Only two methods, Laspeyre's and Paasche's do not satisfy the time reversal test. Besides these two, the other methods of index numbers satisfies the time reversal method.

Fishers index method satisfies time reversal test.

Proof

$$P_{01} = \sqrt{\frac{\Sigma p_1 q_0}{\Sigma p_0 q_0} \times \frac{\Sigma p_1 q_1}{\Sigma p_0 q_1}}$$

If P_{10} is calculated, then the formula would be,

$$P_{10} = \sqrt{\frac{\Sigma p_0 q_1}{\Sigma p_1 q_1} \times \frac{\Sigma p_0 q_0}{\Sigma p_1 q_0}}$$

Time reversal test : $P_{01} \times P_{10} = 1$

$$\sqrt{\frac{\Sigma p_1 q_0}{\Sigma p_0 q_0} \times \frac{\Sigma p_1 q_1}{\Sigma p_0 q_1} \times \frac{\Sigma p_0 q_1}{\Sigma p_1 q_1} \times \frac{\Sigma p_0 q_0}{\Sigma p_1 q_0}} = 1$$

Time reversal test is satisfied by Fisher's index method. Hence proved.

(iii) Factor Reversal Test

According to factor reversal test, when change in price is multiplied with change in quantity, it should give total change in value. That is, if the price of a commodity is increased by 3 times and its quantity has also increased by 4 times, then the total change in value would be 12 times than that of the former value. Thus the formula for commodity whole price and quantity is p_0 and p_1 and q_0 and q_1 for base and current year is,

$$P_{01} \times Q_{01} = \frac{\Sigma p_1 q_1}{\Sigma p_0 q_0}$$

Except Fisher's ideal index, no other method satisfies the factor reversal test.

Proof

$$P_{01} = \sqrt{\frac{\Sigma p_1 q_0}{\Sigma p_0 q_0} \times \frac{\Sigma p_1 q_1}{\Sigma p_0 q_1}}$$

$$Q_{01} = \sqrt{\frac{\Sigma q_1 p_0}{\Sigma q_0 p_0} \times \frac{\Sigma q_1 p_1}{\Sigma q_0 p_1}}$$

Factor reversal test : $P_{01} \times Q_{01} = \frac{\Sigma p_1 q_1}{\Sigma p_0 q_0}$

$$P_{01} \times Q_{01} = \sqrt{\frac{\Sigma p_1 q_0}{\Sigma p_0 q0} \times \frac{\Sigma p_1 q_1}{\Sigma p_0 q_1} \times \frac{\Sigma q_1 p_0}{\Sigma q_0 p_0} \times \frac{\Sigma q_1 p_1}{\Sigma q_0 p_1}} = \sqrt{\frac{\Sigma p_1 q_1}{\Sigma p_0 q_0} \times \frac{\Sigma q_1 p_1}{\Sigma q_0 p_0}}$$

$$= \sqrt{\left(\frac{\Sigma p_1 q_1}{\Sigma p_0 q_0}\right)^2} = \frac{\Sigma p_1 q_1}{\Sigma p_0 q_0}$$

Fisher's ideal index satisfies the factor reversal test. Hence proved.

(iv) Circular Test

According to circular test, if P_{ab} is the price index whose base year is '*a*' and current year is '*b*', P_{bc} is the price index with base year '*b*' and current year is '*c*' and P_{ca} is the price index whose base year is '*c*' and current year is '*a*' then,

$$\boxed{P_{ab} \times P_{bc} \times P_{ca} = 1}$$

The following methods satisfies the circular test,

(a) Simple geometric mean of price relatives

(b) Simple aggregative index

(c) Weighted aggregative index.

Apart from these methods, no other method of price index satisfies the circular test.

PROBLEMS ON TESTS OF CONSISTENCY OF INDEX NUMBER

Q28. Calculate Fisher's Ideal Index Number and test whether it satisfies Time Reversal and Factor Reversal Test for the following data.

Commodity	Base year		Current year	
	Price (₹)	Qty (kg)	Price (₹)	Qty (kg)
A	32	50	30	50
B	30	35	25	40
C	16	55	18	50

Solution : (Model Paper-III, Q10(b) | Oct./Nov.-16, Q13(b) (OU))

Calculation of Fisher's Ideal Index Number

Commodity	p_0	q_0	p_1	q_1	p_1q_0	p_0q_0	p_1q_1	p_0q_1
A	32	50	30	50	1500	1600	1500	1600
B	30	35	25	40	875	1050	1000	1200
C	16	55	18	50	990	880	900	800
					$\Sigma p_1q_0 = 3365$	$\Sigma p_0q_0 = 3530$	$\Sigma p_1q_1 = 3400$	$\Sigma p_0q_1 = 3600$

Fisher's Ideal Index, $P_{01}^F = \sqrt{\dfrac{\Sigma p_1 q_0}{\Sigma p_0 q_0} \times \dfrac{\Sigma p_1 q_1}{\Sigma p_0 q_1}} \times 100$

$$= \sqrt{\frac{3365}{3530} \times \frac{3400}{3600}} \times 100$$

$$= \sqrt{0.953 \times 0.944} \times 100$$

$$= 94.85$$

Time Reversal Test

Time Reversal Test is satisfied when, $P_{01} \times P_{10} = 1$

$$P_{01} = \sqrt{\frac{\Sigma p_1 q_0}{\Sigma p_0 q_0} \times \frac{\Sigma p_1 q_1}{\Sigma p_0 q_1}} = \sqrt{\frac{3365}{3530} \times \frac{3400}{3600}}$$

$$P_{10} = \sqrt{\frac{\Sigma p_0 q_1}{\Sigma p_1 q_1} \times \frac{\Sigma p_0 q_0}{\Sigma p_1 q_0}} = \sqrt{\frac{3600}{3400} \times \frac{3530}{3365}}$$

$$P_{01} \times P_{10} = \sqrt{\frac{3365}{3530} \times \frac{3400}{3600} \times \frac{3600}{3400} \times \frac{3530}{3365}}$$

$$= \sqrt{1} = 1$$

$$\boxed{P_{01} \times P_{10} = 1}$$

∴ This index number satisfies the time reversal test.

Factor Reversal Test

Factor reversal test is satisfied when,

$$P_{01} \times q_{01} = \frac{\Sigma p_1 q_1}{\Sigma p_0 q_0}$$

$$P_{01} = \sqrt{\frac{\Sigma p_1 q_0}{\Sigma p_0 q_0} \times \frac{\Sigma p_1 q_1}{\Sigma p_0 q_1}} = \sqrt{\frac{3365}{3530} \times \frac{3400}{3600}}$$

$$q_{01} = \sqrt{\frac{\Sigma q_1 p_0}{\Sigma q_0 p_0} \times \frac{\Sigma q_1 p_1}{\Sigma q_0 p_1}} = \sqrt{\frac{3600}{3530} \times \frac{3400}{3365}}$$

$$\therefore \quad p_{01} \times q_{01} = \sqrt{\frac{3365}{3530} \times \frac{3400}{3600} \times \frac{3600}{3530} \times \frac{3400}{3365}}$$

$$= \sqrt{\left(\frac{3400}{3530}\right)^2}$$

$$= \frac{3400}{3530} = \frac{\Sigma p_1 q_1}{\Sigma p_0 q_0}$$

$$\boxed{\therefore p_{01} \times q_{01} = \frac{\Sigma p_1 q_1}{\Sigma p_0 q_0}}$$

∴ This index number satisfies the factor reversal test.

Q29. Calculate Fisher's ideal index from the following data and show how it satisfies T.R.T and F.R.T.

Commodities	2010		2012	
	Price	Total Value	Price	Total Value
P	8	80	10	110
Q	10	90	12	108
R	16	256	20	340

Solution : Oct./Nov.-14, Q13(b) (OU)

In the given problem, quantity of each commodity is not given. Therefore, calculate amount of quantity by using following formula,

$$\text{Quantity} = \frac{\text{Total value}}{\text{Price}}$$

Commodity	2010			2012		
	Total value	Price (p_0)	Quantity (q_0)	Total value	Price (p_1)	Quantity (q_1)
P	80	8	10	110	10	11
Q	90	10	9	108	12	9
R	256	16	16	340	20	17

Calculation of Fisher's Ideal Index

Commodity	p_0	q_0	p_1	q_1	p_1q_1	p_0p_0	p_1p_1	p_0p_1
P	8	10	10	11	100	80	110	88
Q	10	9	12	9	108	90	108	90
R	16	16	20	17	320	256	340	272
					$p_1p_0 = 528$	$p_0p_0 = 426$	$p_1p_0 = 558$	$p_0p_1 = 450$

Fisher's Ideal Index,

$$p_0 = \sqrt{\frac{\Sigma p_1q_0}{\Sigma p_0q_0} \times \frac{\Sigma p_1q_1}{\Sigma p_0q_1}} \times 100 = \sqrt{\frac{528}{426} \times \frac{558}{450}} \times 100$$

$$= \sqrt{1.24 \times 1.24} \times 100 = \sqrt{1.5376} \times 100$$

$$= 124$$

Time Reversal Test

Time Reversal Test is satisfied when,

$$p_{01} \times p_{10} = 1$$

$$p_{10} = \sqrt{\frac{\Sigma p_0q_1}{\Sigma p_1q_1} \times \frac{\Sigma p_0q_0}{\Sigma p_1q_0}}$$

$$p_{01} \times p_{10} = \sqrt{\frac{\Sigma p_1q_0}{\Sigma p_0q_0} \times \frac{\Sigma p_1q_1}{\Sigma p_0q_1} \times \frac{\Sigma p_0q_1}{\Sigma p_1q_1} \times \frac{\Sigma p_0q_0}{\Sigma p_1q_0}}$$

$$p_{01} \times p_{10} = \sqrt{\frac{528}{426} \times \frac{558}{450} \times \frac{450}{558} \times \frac{426}{528}}$$

$$= 1$$

Hence, Time Reversal Test is satisfied.

Factor Reversal Test

Factor Reversal Test is satisfied when,

$$p_{01} \times q_{01} = \frac{\Sigma p_1q_1}{\Sigma p_0q_0}$$

$$q_{01} = \sqrt{\frac{\Sigma p_0q_1}{\Sigma p_0q_0} \times \frac{\Sigma p_1q_1}{\Sigma p_1q_0}}$$

$$p_{01} \times q_{01} = \sqrt{\frac{\Sigma p_1q_0}{\Sigma p_0q_0} \times \frac{\Sigma p_1q_1}{\Sigma p_0q_1} \times \frac{\Sigma p_0q_1}{\Sigma p_0q_0} \times \frac{\Sigma p_1q_1}{\Sigma p_1q_0}}$$

$$p_{01} \times q_{01} = \sqrt{\frac{528}{426} \times \frac{558}{450} \times \frac{450}{426} \times \frac{558}{528}}$$

$$p_{01} \times q_{01} = 1.31$$

$$\frac{\Sigma p_1q_1}{\Sigma p_0q_0} = \frac{558}{426} = 1.31$$

$$\therefore \quad p_{01} \times q_{10} = \frac{\Sigma p_1q_1}{\Sigma p_0q_0}$$

Hence, Factor Reversal Test is satisfied.

2.5 BASE SHIFTING

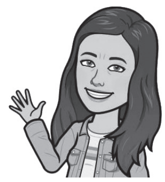
Q30. What is Base Shifting? Explain it with an illustration.

Answer :

Base Shifting

Base Shifting refers to the process of shifting a base period of an index. It is also known as changing of base. It helps in calculating the index numbers based on new base. As the old base gets outdated, it is required to shift or change such old base to new-base.

The base shifting can be calculated by using the following formula,


Keep an eye on me

$$\text{Base Shifting} = \frac{\text{Current year's old index number}}{\text{New base year's old index number}} \times 100$$

Illustration

From the following information of index numbers calculate base shifting.

Consider 2001 base year of old index number and 2004 as new base year.

Years	2001	2002	2003	2004	2005
Index Numbers	100	120	150	180	225

Solution :

The calculation of index number based on new base year is,

$$= \frac{\text{Current year's old index number}}{\text{New base year's old index number}} \times 100$$

Years	Old Index Numbers (Base Year 2001 = 100)	New Index Numbers (Base Year 2004) (2004 = 180)
2001	100	$\frac{100}{180} \times 100 = 55.55$
2002	120	$\frac{120}{180} \times 100 = 66.67$
2003	150	$\frac{150}{180} \times 100 = 83.33$
2004	180	$\frac{180}{180} \times 100 = 100$
2005	225	$\frac{225}{180} \times 100 = 125$

PROBELMS ON BASE SHIFTING

Q31. The following are the indices (2007, Base):

Year	2007	2008	2009	2010	2011	2012	2013	2014	2015	2016	2017
Indices	100	120	122	116	120	120	137	136	149	156	137

Shift the base to 2012 and recast the index numbers.

Solution : (Model Paper-II, Q10(b) | May/June-18, Q10(a) (OU))

The calculation of index number based on new base year is $= \frac{\text{Current year's old index number}}{\text{New base year's old index number}} \times 100$

Years	Old Index Numbers (Base year 2007 = 100)	New Index Numbers (Base year 2012 = 120)
2007	100	$\frac{100}{120} \times 100 = 83.33$
2008	120	$\frac{120}{120} \times 100 = 100$
2009	122	$\frac{122}{120} \times 100 = 101.67$
2010	116	$\frac{116}{120} \times 100 = 96.67$
2011	120	$\frac{120}{120} \times 100 = 100$
2012	120	$\frac{120}{120} \times 100 = 100$
2013	137	$\frac{137}{120} \times 100 = 114.17$
2014	136	$\frac{136}{120} \times 100 = 113.33$
2015	149	$\frac{149}{120} \times 100 = 124.17$
2016	156	$\frac{156}{120} \times 100 = 130$
2017	137	$\frac{137}{120} \times 100 = 114.17$

2.6 SPLICING OF INDEX

Q32. What is Splicing? What are its conditions, importance and types?

Answer :

Splicing

Splicing is the process of combining of two or more overlapping indices in which various bases are converted into a single series. In other words, splicing of index numbers can be defined as the procedure of converting two or more series of index numbers of different bases into a single or continuous series of index numbers.

Conditions of Splicing

The following conditions are essential for splicing the index numbers,

1. Atleast two series of index numbers should be from same group.
2. The base period of different indices are built on different series.
3. Indices of two bases should be there in a year. For example, in 2015 there may be two index numbers with 2005 and another with 2013.

Importance of Splicing

The importance of splicing of index numbers is highlighted in the following points,

1. It is important in the situation where a series of index numbers with old base is discontinued and new series of index numbers is constructed with new base year.
2. It helps to bring two or more disjoint index series under a common base.
3. It helps to maintain continuous series in index numbers.
4. It helps to compare the index numbers of various years.

Types of Splicing

Generally, splicing is categorized into two types. They are as follows,

1. Backward splicing
2. Forward splicing.

1. Backward Splicing

Backward splicing is used for splicing a new series of indices to continue with old series of indices. The formula used for backward splicing is as follows,

$$\text{Backward Splicing} = \frac{\text{Index A of Current Year}}{\text{Index A of Common Year}} \times 100$$

2. Forward Splicing

Forward splicing is used for splicing old series of indices to continue with new series of indices. The formula used for forward splicing is as follows,

$$\text{Forward Splicing} = \frac{\text{Index B of Current Year} \times \text{Index A of Common Year}}{100}$$

PROBLEMS ON SPLICING OF INDEX

Q33. The following table gives two series of index numbers with 2005 and 2010 as base year. Obtain a continuous series of index number by considering 2010 as base year.

Years	2005	2006	2007	2008	2009	2010	2011	2012	2013	2014
Index No. (2005 = 200)	200	240	250	280	300	320				
Index No. (2010 = 200)						200	220	240	260	280

Solution :

$$\text{Backward Splicing} = \frac{\text{Index A of Current Year}}{\text{Index A of Common Year}} \times 100$$

Years	Index Number (2005 = 200)	Index Number (2010 = 200)	Backward Splicing
2005	200		$\frac{200}{320} \times 100 = 62.5$
2006	240		$\frac{240}{320} \times 100 = 75$
2007	250		$\frac{250}{320} \times 100 = 78.125$
2008	280		$\frac{280}{320} \times 100 = 87.50$
2009	300		$\frac{300}{320} \times 100 = 93.75$
2010	320	200	$\frac{320}{320} \times 100 = 100$
2011		220	220
2012		240	240
2013		260	260
2014		280	280

Q34. The following table gives two series of index numbers with 2006/2010 as base year. Obtain a combined series of index numbers with 2006 as base year.

Years	2006	2007	2008	2009	2010	2011	2012	2013	2014
Index No. (2006 = 100)	100	105	118	120	125				
Index No. (2010 = 100)					100	120	132	145	160

Solution :

$$\text{Forward Splicing} = \frac{\text{Index B of Current Year} \times \text{Index A of Common Year}}{100}$$

Years	Index No. (2006 = 100) Series A (Forward Splicing)	Index No. (2010 = 100) Series B
2006	100	
2007	105	
2008	118	
2009	120	
2010	125	100
2011	$\frac{120 \times 125}{100} = 150$	120
2012	$\frac{132 \times 125}{100} = 165$	132
2013	$\frac{145 \times 125}{100} = 181.25$	145
2014	$\frac{160 \times 125}{100} = 200$	160

2.7 DEFLATING OF INDEX

Q35. What do you mean by deflating? Explain with an example.

Answer :

Deflating

Deflating refers to the process of making allowances for the impact of changing prices. A rise in price level means a reduction in the purchasing power of money.

Example

The price of rice in 2010 was ₹ 28 per kg but in 2013 the price increased to ₹ 45 per kg. It means a person who could buy 1 kg rice for ₹ 28 in 2010 would be able to buy only half kg rice in 2013 as their income level remains same. During the period of inflation, the purchasing power of money is the reciprocal of the price index. This reciprocal relationship can be shown in the form of formula as follows,

$$\text{Purchasing power of money} = \frac{1}{\text{Price index}}$$

Suppose, if price of a commodity 30%, price index is 1.30 and the purchasing power of rupee is 1/1.30 = 0.77 or 77 paise. As value of money decreases with the increase in prices wage workers or salaried people show more interest in real wage rather than money wage. Real waged can be obtained by using the following formulae,

$$\text{Real wage} = \frac{\text{Money wage}}{\text{price index}} \times 100$$

$$\text{Real wage or income index No.} = \frac{\text{Index of money wages}}{\text{Consumer price index}}$$

PROBLEM ON DEFLATING OF INDEX

Q36. The annual income of a employee and the general index numbers of price during 2011-2019 is given in the following table. Prepare index number to show the changes in real income of the employee and comment on price increase,

Year	2011	2012	2013	2014	2015	2016	2017	2018	2019
Income	3,500	4,000	5,300	5,500	6,400	6,000	7,200	7,500	6,800
Price Index No.	100	120	145	160	250	320	450	530	600

Solution :

Calculation of index Numbers Showing Changes in the Real income of the Employee

Years (1)	Income (₹) (2)	Price Index No. (3)	Real Income (4) = [2÷3]× 100	Real income Index No. (5)
2011	3,500	100	$\frac{3,500}{100} \times 100 = 3,500$	3,500/3,500 × 100=100
2012	4,000	120	$\frac{4,000}{120} \times 100 = 3,333.33$	3,333/3,500 ×100 = 95.23
2013	5,300	145	$\frac{5,300}{145} \times 100 = 3,655.17$	3,655.17/3,500 ×100 = 104.43
2014	5,500	160	$\frac{5,500}{160} \times 100 = 3,437.5$	3,437.5/3,500 × 100 = 98.2
2015	6,400	250	$\frac{6,400}{250} \times 100 = 2,560$	2,560 / 3,500 × 100 =73.14
2016	6,000	320	$\frac{6000}{320} \times 100 = 1,875$	1,875/3,500 × 100 =53.57
2017	7,200	450	$\frac{7,200}{450} \times 100 = 1,600$	1,600/3,500 ×100 = 45.71
2018	7,500	530	$\frac{7,500}{530} \times 100 = 1,415.09$	1,415.09/3,500 × 100 = 40.43
2019	6,800	600	$\frac{6,800}{600} \times 100 = 1,133.3$	1,133.3/3,500 × 100 = 32.38

2.8 CONSUMER PRICE INDEX

Q37. What are consumer price index numbers? Explain the steps in their construction and list out their uses.

Answer :

Consumer Price Index Numbers

Consumer price index numbers also known as cost of living index numbers are used to measure the purchasing power of a particular class of people in relation to the changes in retail prices. In other words, it studies how price variations effect the cost of living or purchasing power of a group of people.

While constructing cost of living index number, a particular section of society is selected like rich, middle, poor and a study is conducted to know how price variations effect the consumption levels of that section. Based on such information, Consumer Price Index Numbers (CPIN) is constructed.

Steps in Construction of Consumer Price Index Numbers (CPIN)

The steps in construction of consumer price index numbers are as follows,

1. Selection of Group of People

A group of people or class of people is selected to construct CPIN. Apart from class of people, the area (i.e., rural or urban, city or town) should be clearly specified. The group of people selected for constructing CPIN must be homogenous to a maximum extent.

2. Conducting Family Budget Enquiry

An enquiry of family budget is conducted to know how much money an average family spends on the consumption of different items. These items are broadly categorized into five groups namely,

(a) Food

(b) Clothing

(c) Fuel and lighting

(d) House Rent and

(e) Miscellaneous.

Each of the above groups is further sub- categorised into small groups, Example, The group "food" is subdivided into cereals (like wheat, rice, pulses and so on) meat, fish , milk, fruits, vegetables and so on.

3. Price Quotations

While gathering information about retail prices a proper care should be taken as retail prices varies from place to place and from shop to shop. Information about retail prices should be gathered from those local markets where selected class of people are located.

Uses of CPIN

The uses of CPIN are as follows,

1. Consumer Price Index Numbers (CPIN) are used in the preparation of wage contracts and wage negotiations.
2. They assists the government and business organization in deciding Dearness Allowance [D.A] to be paid to their employees.
3. They are used for deflating income and value series in National Income of the country.
4. They are used to measure purchasing power of money.
5. They assist in calculating Real wage by considering the variations in money income and price level.

Q38. Explain the methods in construction of consumer price index.

Answer :

Methods of Construction of Consumer Price Index Numbers

It was observed that the significance of consumption items is different for different groups of people. Even people belonging to same class might have different opinion regarding the significance of consumption items.

This is the reason why cost of living index is determined as weighted indices by taking into account the relative significance of consumption items. The significance of consumption items is determined based upon the money spent by people on various items. Further cost of living index numbers are constructed by adopting the following two methods,

1. Aggregate Expenditure Method/Weighted Aggregate Method

Under this method the quantities (q) consumed in the base year are taken as weights. It can be expressed as,

$$\text{Consumer Price Index} = \frac{\text{Total expenditure in current year}}{\text{Total expenditure in base year}}$$

$$= \frac{\sum p_1 q_0}{\sum p_0 q_0} \times 100$$

Where,

p_1 = Current year price

p_0 = Base year price

q_0 = Base year quantity.

2. Family Budget Method/Method of Weighted Relatives

Under this method weighted average of price relatives is calculated to obtain cost of living index. Where, weights are equal to the quantities consumed in the base year. It can be expressed as,

$$\text{Consumer price index} = \frac{\Sigma WI}{\Sigma W}$$

Where, I = Price relative = $\frac{p_1}{p_0} \times 100$ and $w = p_0 q_0$,

By substituting the values of W and I, the following is obtained,

$$\text{Consumer Price Index} = \frac{\sum p_0 q_0 \left[\frac{p_1}{p_0} \times 100\right]}{\sum p_0 q_0}$$

$$= \frac{\sum p_1 q_0}{\sum p_0 q_0} \times 100$$

PROBLEM ON CONSUMER PRICE INDEX

Q39. Calculate cost of living index from the following data:

Article	Price in Base year (₹)	Price in Current year (₹)	Quantity in Base year (₹)
A	6	8	50
B	2	3	100
C	5	6	60
D	10	12	30

Solution : May/June-18, Q3(b) (KU)

Calculation of Cost of Living Index Numbers by Aggregate Expenditure Method

Article	Prices In (₹)		Aggregate Expenditure		
	Base year [P_0]	Current year [P_1]	Base Year q_0	$P_1 q_0$	$P_0 q_0$
A	6	8	50	400	300
B	2	3	100	300	200
C	5	6	60	360	300
D	10	12	30	360	300
				$\Sigma P_1 q_0$= 1,420	$\Sigma P_0 q_0$= 1,100

$$\text{Cost of Living Index Number} = \frac{\Sigma p_1 q_0}{\Sigma p_0 q_0} \times 100 = \frac{1,420}{1,100} \times 100$$

$$= 129.09$$

Calculation of Living Index Numbers by Family Budget Method

Article	Prices In (₹)		Quantity	Price Relative (1)	W = Value in Base (2)	
	Base year $[P_0]$	Current year $[P_1]$	Base year (q_0)	$P = P_1/P_0 \times 100$	$W = P_0 q_0$	IW
A	6	8	50	8/6 × 100 = 133.33	300	39,999
B	2	3	100	3/2 × 100 = 150	200	30,000
C	5	6	60	6/5 × 100 = 120	300	36,000
D	10	12	30	12/10 × 100 = 120	300	36,000
					ΣW= 1,100	ΣIW=1,41,999

$$\text{Cost of Living Index Number} = \frac{\Sigma IW}{\Sigma W}$$

$$= \frac{1,41,999}{1,100}$$

$$= 129.09$$

EXERCISE AND PRACTICE QUESTIONS

SHORT QUESTIONS

THEORY

Q1. Define Index Numbers. [Refer, Q1]

Q2. Importance of Index Numbers. [Refer, Q2] May/June-18, Q2 (OU)

OR

Advantages of Index Numbers March/April-17, Q8 (OU)

OR

Explain the uses of Index Numbers. March/April-14, Q7 (OU)

Q3. Types of Index Numbers [Refer, Q3] (May/June-19, Q2 (OU) | Oct./Nov.-14, Q4 (OU))

Q4. Marshall Edge Worth Method. [Refer, Q4] May/June-19, Q2 (MGU)

Q5. What is "Cost of living Index"? [Refer, Q5] May/June-18, Q1(b) (KU)

PROBLEMS

Q6. Calculate index number by simple aggregative method. [Refer Similar, Q6]

	A	B	C	D
Price in 2005 (₹)	324	512	514	264
Price in 2007 (₹)	342	328	378	290

(Ans: P_{01} = 165.8)

Q7. The prices for period 2006 to 2008 are given below:

Year	Price
2006	50
2007	52
2008	54

Calculate simple price index taking 2016 as base year. [Refer Similar, Q7]

(Ans: $P_{2006,\,2006}$ = 100%; $P_{2006,\,2007}$ = 104%; $P_{2006,\,2008}$ = 108%)

Q8. From the following data calculate a price index based on price relatives method using Arithmetic Mean. [Refer Similar, Q8]

Commodity	A	B	C	D	E	F
Price 2015 (₹)	90	120	40	100	170	240
Price 2016 (₹)	110	140	60	150	180	260

(Ans: P_{01} = 251.034)

Q9. Calculate Index number by Average Price Relative Method by using Arithmetic Mean. [Refer Similar, Q9]

Commodity	P	Q	R	S	T
Price 2019	4	12	20	10	24
Price 2020	8	16	30	10	16

(Ans: 260)

Q10. Calculate index number for the following data by arithmetic average of price relatives method. [Refer Similar, Q8]

Commodities	A	B	C	D	E	F
Price in 1980	40	60	20	50	80	100
Price in 1990	50	60	30	70	90	110

March/April-16, Q7 (OU)

(Ans: P_{01} = 122.916).

ESSAY QUESTIONS

THEORY

Q11. Define Index Number. What are its features and uses? [Refer, Q11] May/June-19, Q10(a) (OU)

OR

What is Index Number? Write importance of index numbers. May/June-18, Q3(a) (KU)

Q12. What is the importance and limitations of index numbers? Explain. [Refer, Q12] May/June-19, Q7(b) (MGU)

Q13. Discuss the problems in the construction of Index Numbers. [Refer, Q14]

Q14. What is Splicing? What are its conditions, importance and types? [Refer, Q32]

Q15. What are consumer price index numbers? Explain the steps in their construction and list out their uses. [Refer, Q37]

PROBLEMS

Q16. Compute a price index from the following data,

(a) Simple aggregative method

(b) Average of price relative method by using Arithmetic mean and Geometric mean.
[Refer Similar, Q6, Q9]

Commodity	A	B	C	D	E	F
Price in 2010 (₹)	10	20	30	40	50	25
Price in 2015 (₹)	15	25	30	40	55	35

(Ans: (a) P_{01} = 114.29 (b) 120.83).

Q17. From the following data, calculate Laspeyre's, Paache's, Dorbish and Bowley, Marshall Edgeworth methods and Fisher's Ideal Index. [Refer Similar, Q21]

Item	Base Year		Current Year	
	Price(₹)	Qty(KG)	Price(₹)	Qty(KG)
A	6	50	10	56
B	2	100	2	120
C	4	60	6	60
D	10	30	12	24
E	8	40	12	36

(Ans: 139.71; 139.88; 139.8; 139.79; 139.796).

Q18. Calculate Fisher's ideal index from the following data and show how it satisfies Time Reversal Tests. [Refer Similar, Q28]

Items	Price		Quantity	
	2010	2011	2010	2011
A	8	20	50	60
B	2	6	15	10
C	1	2	20	25
D	2	5	10	8
E	1	5	40	30

(Ans: 266.61).

Q19. Compute Fisher's Ideal Index number and show how its satisfies the Time Reversal and factor reversal tests. [Refer Similar, Q28]

Commodities	Base year 2011		Current year 2018	
	Price	Qty	Price	Qty
P	10	100	20	140
Q	8	150	8	200
R	12	120	18	160
S	20	80	30	80
T	16	160	24	200

(Ans: 148.84).

Q20. Calculate the index number using both the aggregate expenditure method and family budget method for the year 2013 with 2000 as the base year from the following data. [Refer Similar, Q39]

Commodity	Qty in Units 2000	Price P.U 2000 (₹)	Price P.U 2013 (₹)
A	100	8.00	12.00
B	25	6.00	7.50
C	10	5.00	5.25
D	20	48.00	52.00
E	25	15.00	16.50
F	30	9.00	27.00

(Ans: 142.1).

Q21. Compute price index and quantity index numbers for the year 2010 with 2005 as base year using,

(i) Laspeyer's method

(ii) Paasche's method. [Refer Similar, Q23, Q26]

Commodity	Quantity (units)		Value (₹)	
	2005	2010	2005	2010
A	100	150	500	900
B	80	100	320	500
C	60	72	150	360
D	30	33	360	297

March/April-13, Q13(a) (OU)

(Ans: (i) Laspeyer's Price Quantity Indices

P_{01} = 118.04 , Q_{01} = 129.77

(ii) Paasche's Price and Quantity Indices

P_{01} = 119.18 , Q_{01} = 131.02)

Q22. Calculate Fisher's Ideal Index number from the data: [Refer Similar, Q24]

Commodities	2005		2010	
	Price Per Unit (₹)	Total Expenditure (₹)	Price Per Unit (₹)	Total Expenditure (₹)
A	8	80	10	120
B	10	120	12	96
C	5	40	5	50
D	4	56	3	60
E	20	100	25	150

March/April-16, Q13(a) (OU)

(Ans: P_{01} = 112.8).

Q23. Compute Laspyre's and Paasche's index numbers from the following data: [Refer Similar, Q25]

	Base Year		Current Year	
	Price (₹)	Expenditure	Price (₹)	Expenditure
P	2	40	5	75
Q	4	16	8	40
R	1	10	2	24
S	5	25	10	60

Sept./Oct.-15, Q13(b) (OU)

(Ans: Laspyre's Index Method p_{01} = 221.98 ; Paasche's Index Method p_{01} = 216.30).

Q24. The following are the index numbers of a commodity taking 2011 as the base:

Years	2011	2012	2013	2014	2015
Index Numbers	200	240	300	360	450

Find the index numbers by changing the base to 2013. [Refer Similar, Q30]

(Ans: 66.67, 80, 100, 120, 150).

INTERNAL ASSESSMENT/EXAM

I Multiple Choice

1. __________ is a numerical value characterizing the change in complex economic phenomena over a period of time or space. [ ]

   (a) Current value (b) Base value

   (c) Index number (d) Topic number

2. $\frac{\text{Current year}}{\text{Base year}} \times 100$ is the formula of ___________. [ ]

   (a) Multiple Price index (b) Double Price Index

   (c) Single – Double Price Index (d) Single Price Index

3. Laspeyre's Index Method was introduced by ___________. [ ]

   (a) Etiebne Laspeyres (b) Herman Laspeyres

   (c) Shane Laspeyres (d) Angelo Laspeyres

4. $P_{01} = \frac{\Sigma p_1 q_0}{\Sigma p_0 q_0} \times 100$ is the formula of ________. [ ]

   (a) Paasche's Index Method (b) Laspeyre's Price Index Method

   (c) Fishers Ideal Index Method (d) Marshall - Edgeworth Index Method

5. In _________ index method, prices are weighted by the quantity consumed in the current year, rather than considering quantity consumed in base year. [ ]

   (a) Laspeyre's (b) Fishers

   (c) Paasche's (d) Marshall-Edgeworth's

6. In ___________ index method, both the current year as well as base year quantities are considered as to calculate the index. [ ]

   (a) Marshall-Edge worth's (b) Fishers

   (c) Paasche's (d) Laspeyre's

7. Marshall-Edgeworth's index method was proposed by ___________ mathematicians. [ ]

   (a) 3 (b) 2

   (c) 1 (d) 6

8. There are ______ tests of consistency of index numbers. [ ]

   (a) 1 (b) 2

   (c) 3 (d) 4

9. $\frac{\text{Current year's old index number}}{\text{New base year's old index number}} \times 100$ is the formula of __________. [ ]

   (a) Calculation of index number based on new base year

   (b) Calculation of index number based on old base year

   (c) Calculation of index number based on current base year

   (d) None of the above

10. _________ is used for splicing old series of indices to continue with new series of indices. [ ]

   (a) Base shifting (b) Forward splicing

   (c) Backward splicing (d) None of the above

II Fill in the Blanks

1. __________ are used to measure the changes that takes place in the total value of the variable.
2. __________ measures the variation in the volume of goods manufactured or consumed or distributed in quantitative terms.
3. __________ are used for measuring differences in the magnitude of a group of related variables.
4. Index numbers are __________ by which changes in the magnitude of a phenomenon are measured from time to time or place to place.
5. __________ test is basically used for checking whether the selected method would work both backward or forward or not.
6. According to _________ test, when change in price is multiplied with change in quantity, it should give total change in value.
7. Index numbers are classified into ________ types.
8. In value index numbers, value = _________.
9. Formula of Marshall-Edgeworth's index method is ___________.
10. _________ refers to the process of shifting a base period of an index.

KEY

I. Multiple Choice

1. (c)
2. (d)
3. (a)
4. (b)
5. (c)
6. (a)
7. (b)
8. (d)
9. (a)
10. (b)

II. Fill in the Blanks

1. Value index numbers
2. Quantity index numbers
3. Index numbers
4. Series of numbers
5. Time reversal
6. Factor reversal
7. 3
8. Price × Quantity
9. $P_{01} = \dfrac{\Sigma p_1 q_0 + \Sigma p_1 q_1}{\Sigma p_0 q_0 + \Sigma p_0 q_1} \times 100$
10. Base shifting.

III Very Short Questions and Answers

Q1. Define Index Numbers.

Answer :

According to Maslow, "Index number is a numerical value characterizing the change in the complex economic phenomena over a period of time or space".

Q2. What are the types of index numbers?

Answer :

Following are the types of index numbers,

1. Price index numbers
2. Quantity index numbers and
3. Value index numbers.

Q3. Write briefly about Laspeyre's Index Method.

Answer :

Laspeyre's index method was introduced by a famous statistician named "Laspeyre's". In this method, prices of all items or commodities are weighted by the quantity, consumed both in the base and the current year.

Q4. What is Unit Test?

Answer :

According to unit test, the formula of index number should be independent of the units under which prices and quantities are measured.

Q5. What is Deflating?

Answer :

Deflating refers to the process of making allowances for the impact of changing prices. An increase in price level leads to a decrease in purchasing power of money.

UNIT 3

Time Series

SYLLABUS

Introduction - Components – Methods-Semi Averages - Moving Averages – Least Square Method - Deseasonalisation of Data – Uses and Limitations of Time Series.

LEARNING OBJECTIVES

- ✓ *Concept of Time Series Analysis.*
- ✓ *Components of Time Series.*
- ✓ *Methods such as Semi Averages, Moving Averages and Least Square Method.*
- ✓ *Concept of Deseasonalisation of Data.*
- ✓ *Uses and Limitations of Time Series.*

INTRODUCTION

Time series is an arrangement of statistical data in a chronological order i.e., in accordance with the time of its occurrence. Secular trend or long term movements, short term fluctuations or periodic movements and random or erratic fluctuations are the components of time series. The analysis of time series is not only used by the economist and business but it is also followed and used by the scientist, astronomist, geologist, sociologist, biologist and researchers.

The various methods which are used for measuring trend component of Time Series are Semi-Average Method, Moving Average Method and Least Square Method. Deseasonalisation of data refers to the process of eliminating seasonal fluctuations from the given time series for, it facilitates in adjusting the given time series for seasonal fluctuations and therefore left out with variables like trend component, cyclical and irregular variations.

PART-A SHORT QUESTIONS AND ANSWERS

Q1. What is Time Series?


I am Simple and Easy

Answer : Model Paper-III, Q4

" A time series may be defined as a collection of readings belonging to different time periods, of some economic variable or composite of variables".

– *Ya-Lun Chou*

Time series refers to the arrangement of statistical data in chronological order i.e., according to the time of occurrence. It represents the changing moments of variables over a particular period of time.

Time series plays an important role in business and economics. So, Economists developed many statistical techniques for analyzing time series data. However, these techniques can also be applied to study time series of other disciplines which are not related to economics and statistics like natural sciences, social sciences etc.

The functional relationship of time series can be mathematically represented as,

$$y = f(t)$$

Where, y = variable under consideration

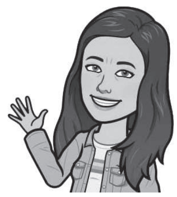
f = functional relationship

t = times $t_1, t_2, t_3,.... t_n$.

Q2. Utility of Time Series Analysis. (Model Paper-I, Q4 | May/June-19, Q4 (OU))

OR

What are the uses of time series? May/June-18, Q4 (OU)


Keep an eye on me

Answer :

The utility or uses of time series analysis is as follows,

1. It Helps in Understanding Past Behaviour

It helps in understanding the past behaviour by considering the changes that have taken place in the past. With the help of past data they predict the future behaviour.

2. It Helps in Planning Future Operations

It helps in future planning by forecasting the events and their relationship. If regular occurrence of any event is there over a long period then such event is considered to predict the future.

3. It Helps in Evaluating Current Accomplishments

It helps in evaluating the performance by comparing the actual performance with the expected performance and analyze the reasons of variations if any.

4. It Facilitates Comparison

The time series for different periods are compared and various conclusions are drawn upon it. But it is not necessary that everyone should believe on it. As statisticians are not foretellers, they cannot predict 100% accurate results for future events.

Q3. Objectives of Time Series

Answer : May/June-19, Q3 (MGU)

There are two main objectives of time series which are as follows,

1. Studying Past Behaviour of Data

One of the important objective of time series analysis is to identify the patterns in the historical data and isolate the effects of various forces or factors. This helps in predicting the value of variable and also planning for future.

2. Reviewing and Evaluating of a Plan

Time series data forms the basis for review and evaluation of a plan. For example, a company may uses time series analysis in its evaluation policy of controlling inflation which is carried out by studying various price indices.

Q4. What is seasonal variations? Write three features of seasonal variations.

Answer : (Model Paper-II, Q4 | May/June-18, Q1(c) (KU))

Seasonal Variations

Seasonal fluctuations are those periodic movements which occur regularly every year and have their origin in the nature of the year itself. Seasonal variations occur in regular and periodic manner over a span of less than a year, i.e., during a period of twelve months and have the same or almost the same pattern year after year. The Causes of Seasonal Variations are Climate in its widest sense (natural causes) and Customs, habits, conventions (man made causes).

Features of Seasonal Variations

Following are the features of seasonal variations,

1. It indicates a variation which demonstrates periodical pattern of change in time series within a year.
2. It occurs for short repetitive calender period.
3. It may occurs because of variations in temperature, rainfall, public holidays etc.

Q5. What are the Components of Time Series?

Answer :

The components of time series are as follows,

1. Secular Trend (or) Long Term Movements

Trend is the irreversible movement in a time series which continues in general in the same direction over a long period of time i..e, secular trend refers to the general tendency of the time series data to increase or decrease over a long period of time. Trend refers to only smooth, regular, long term movement of the data and has nothing to do with sudden and erratic movements either in upward or downward direction.

2. Seasonal Fluctuations

Seasonal fluctuations are those periodic movements which occur regularly every year and have their origin in the nature of the year itself. Seasonal variations occur in regular and periodic manner over a span of less than a year, i.e., during a period of twelve months and have the same or almost the same pattern year after year. The Causes of Seasonal Variations are Climate in its widest sense (natural causes) and Customs, habits, conventions (man made causes).

3. Cyclical Fluctuations

The wave like movements in a time series with period of fluctuations more than one year are called cyclical variations. Cyclical fluctuations generally exhibit semi-regular periodicity as they are neither as regular as the seasonal variations nor as accidental as the erratic fluctuations.

4. Random Erratic Fluctuations

In addition to the influence of long term and short term forces, every time-series is subjected to occasional influences, which may occur just once, or several times, but without any pattern or regularity. These variations are called random or irregular or erratic fluctuations. A random variation may last a day or last many months.

Q6. Fit a trend line to the following data by the Freehand Method.

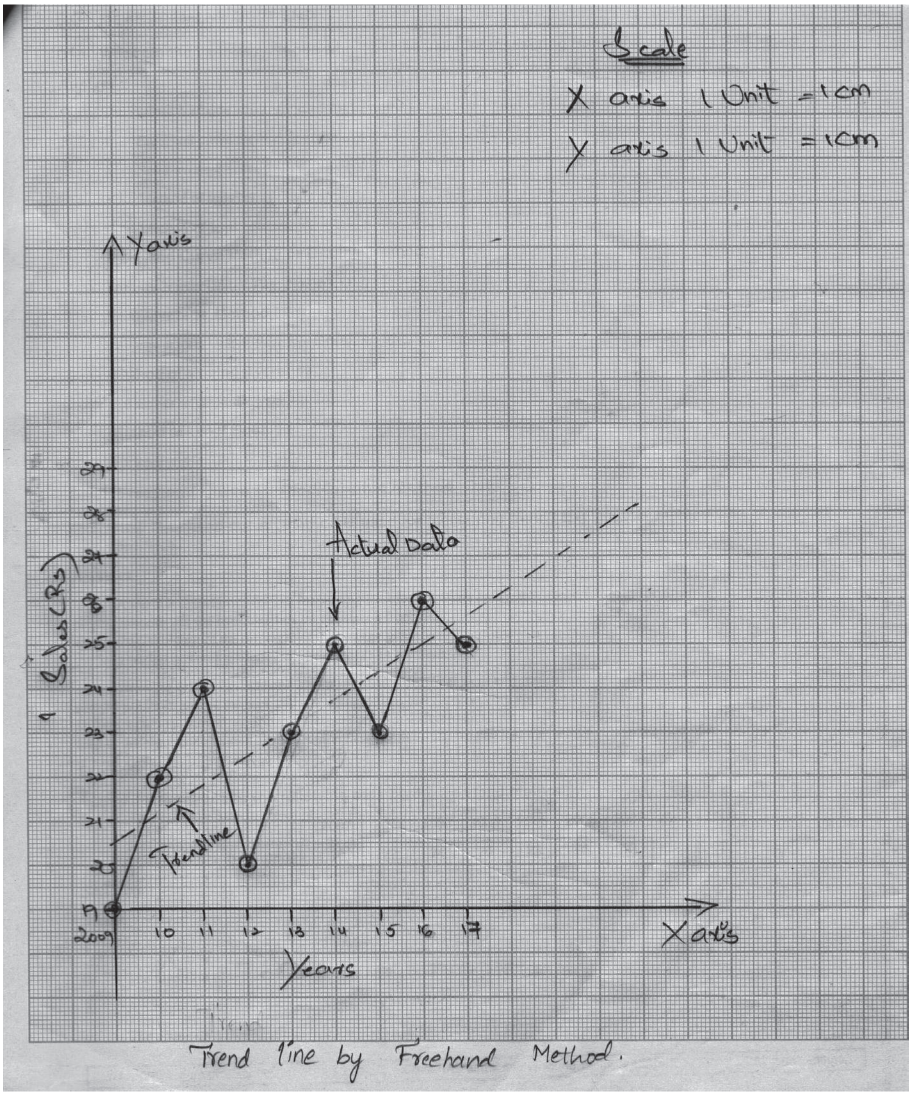
Year	2009	2010	2011	2012	2013	2014	2015	2016	2017
Sales (₹)	19	22	24	20	23	25	23	26	25

Answer : (Model Paper-II, Q5 | May/June-19, Q5 (OU))

The given information is summarized below with graph.

1. Time series data is plotted on the graph
2. The direction of the trend is examined on the basis of the plotted data (dots)
3. A straight line is drawn which shows the direction of the trend.

The actual data and the trend line are shown in the following graph.



Q7. Annual trend of Milk Consumption (Y) is 18.6 + 1.8X. Convert the equation into monthly basis.

Answer : March/April-17, Q7 (OU)

Given that,

$$Y = 18.6 + 1.8X$$

The annual trend equation $Y_e = a + bX$

The formula for converting annual trend into monthly trend is as follows,

$$Y_e = \frac{a}{12} + \frac{b}{12 \times 12} X$$

Substituting the values in the above equation, we get,

$$Y_e = \frac{18.6}{12} + \frac{1.8}{144} X$$

$$Y_e = 1.55 + 0.0125X$$

Q8. Calculate 3 yearly moving averages from the following data,

Year	2007	2008	2009	2010	2011	2012	2013
Sales (in million ₹)	41	44	45	46	47	48	49

Answer : March/April-15, Q7 (OU)

Calculation of 3-Yearly Moving Averages

Year	Sales (in Million ₹)	3-Yearly Total (in Million ₹)	3-Yearly Moving Average
2007	41	–	–
2008	44	41 + 44 + 45 = 130	$\frac{130}{3} = 43.33$
2009	45	44 + 45 + 46 = 135	$\frac{135}{3} = 45$
2010	46	45 + 46 + 47 = 138	$\frac{138}{3} = 46$
2011	47	46 + 47 + 48 = 141	$\frac{141}{3} = 47$
2012	48	47 + 48 + 49 = 144	$\frac{144}{3} = 48$
2013	49	–	–

Q9. Given the following equation $Y_e = 210 + 1.5X$. Time origin is 2006. Time unit is one year, shift the origin to 2011.

Answer : (Model Paper-III, Q5 | Oct./Nov.-14, Q8 (OU))

Given that,

$Y_e = 210 + 1.5X$

Origin of year 2006 is to be shifted to year 2011. Time unit in 1 year, [2006 – 2011 = 5 years]

$2006 = X$

$2007 = X + 1$

$2008 = X + 2$

$2009 = X + 3$

$2010 = X + 4$

$2011 = X + 5$

$Y_e = 210 + 1.5X$

$Y_e = a + b\,(X + K)$

$= 210 + 1.5\,(X + 5)$

$= 210 + 1.5X + 7.5$

$= 217.5 + 1.5X$

PART-B ESSAY QUESTIONS AND ANSWERS

3.1 TIME SERIES – INTRODUCTION – COMPONENTS

Q10. What is Time Series? Explain the components of time series.

Answer :

Time Series

" A time series may be defined as a collection of readings belonging to different time periods, of some economic variable or composite of variables".

– Ya-Lun Chou

Time series refers to the arrangement of statistical data in chronological order (i.e.,) according to the time of occurrence. It represents the changing moments of variables over a particular period of time.

Time series plays an important role in business and economics. So, economists developed many statistical techniques for analyzing time series data. However, these techniques can also be applied to study time series of other disciplines which are not related to economics and statistics like natural sciences, social sciences, etc.

The functional relationship of time series can be mathematically represented as,

$y = f(t)$

Where, y = Variable under consideration

f = Functional relationship

t = Times $t_1, t_2, t_3, ... t_n$

Components of Time Series

The components of time series are as follows,

1. Secular Trend (or) Long Term Movements

Trend is the irreversible movement in a time series which continues in general in the same direction over a long period of time i..e, secular trend refers to the general tendency of the time series data to increase or decrease over a long period of time. Trend refers to only smooth, regular, long term movement of the data and has nothing to do with sudden and erratic movements either in upward or downward direction.

Example

(a) Upward trend : Price, incomes, population

(b) Downward trend: Deaths due to epidemics, bank interest rate, inflation rate.

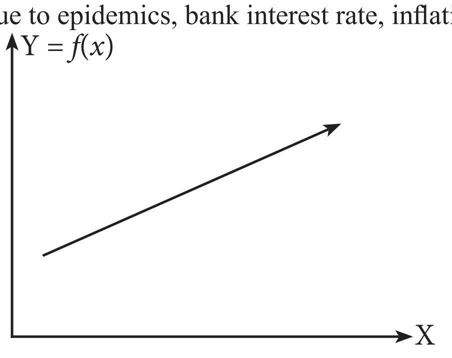


Figure: Secular Trend/Long Term Movements

2. Seasonal Fluctuations

Seasonal fluctuations are those periodic movements which occur regularly every year and have their origin in the nature of the year itself. Seasonal variations occur in regular and periodic manner over a span of less than a year, i.e., during a period of twelve months and have the same or almost the same pattern year after year. The Causes of Seasonal Variations are Climate in its widest sense (natural causes) and Customs, habits, conventions (man made causes).

Example

Time series influenced by seasonal variations are time series relating to agricultural production, sales of agricultural produce, bank deposits, sales and profits in a departmental store.

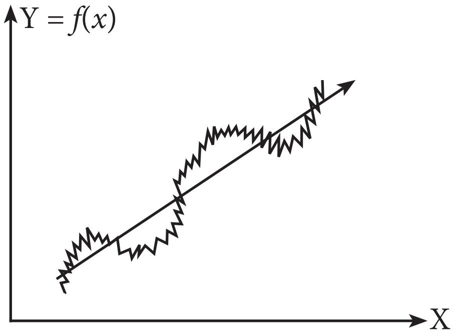


Figure: Seasonal Fluctuations

3. Cyclical Fluctuations

The wave like movements in a time series with period of fluctuations more than one year are called cyclical variations. Cyclical fluctuations generally exhibit semi-regular periodicity as they are neither as regular as the seasonal variations nor as accidental as the erratic fluctuations.

Example

Times of prosperity, production, sales, employment and other economic activity are high, in times of depression it is low.

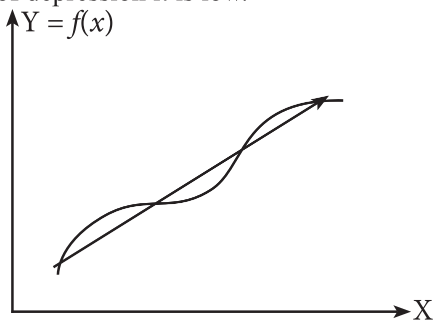


Figure: Cyclical Fluctuations

4. Random Erratic Fluctuations

In addition to the influence of long term and short term forces, every time-series is subjected to occasional influences, which may occur just once, or several times, but without any pattern or regularity. These variations are called random or irregular or erratic fluctuations. A random variation may last a day or last many months.

Example

Wars, earthquakes, floods, fires, strikes, lockouts etc.

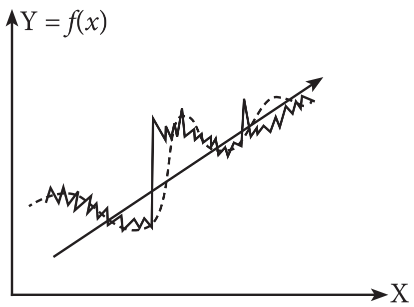


Figure: Random/Erratic Fluctuations

Q11. Write the features of tendency.

Answer : (Model Paper-I, Q11(a) | May/June-18, Q4(a) (KU))

The term trend means "Tendency". A trend which is also called as secular trend refers to the general tendency of the time series data to increase or decrease over a long period of time.

The following are the features of a trend/ tendency,

1. Long Period of Time

It is necessary to collect data for fairly long period of time in order to analyze a trend. Data points related to some period of time cannot be treated as a trend. But, the duration of the period mainly depends on the nature of the data. For example, in order to analyze the trend associated with economic development of a country, data such as population growth, agricultural production, software exports etc are required for at least 8 to 10 years. However, in case of readings taken in every 15 seconds over a period of 10 hours, may sufficient to study the effect of a medicine which regulates human pulse rate.

2. Upward, Downward or Stable Trend

Generally, the secular trend of a series is either upward or downward in nature. For example, data related to sales, production, income, population etc shows an upward trend. Similarly, data related to death rate, people below poverty line etc will shows a downward trend. However, for indices of stock market, a long term trend is upward, downward or stable over a period of time.

3. Impact of Stable Factors

The long term trend facilitates in capturing the impact of some forces which are less or more stable. Such factors fluctuate steadily and continuously over a period of time. For example, factors such as change in people's taste and customs, new material discovery, technology changes and so on. These factors do not reflect any sudden changes.

4. Linear or Non-Linear Trend

A trend can be either a linear or non linear when the time period is plotted on x-axis and value of variable on y-axis. A trend is said to be a linear if rate of increase or decrease is constant. Similarly, a trend can be non-linear or curvi-linear if the rate of growth is uneven or unpredictable. Thus, the plotted points on the graph do not result in a straight line.

3.2 METHODS OF MEASURING TREND

Q12. What are the various methods which can be used for measuring trend component of time series? Explain how trend is measured by using freehand curve with an example.

Answer :

Methods for Measuring Trend

The various methods which are used for measuring trend component of time series are,

1. Free hand curve method
2. Semi average method
3. Moving average method
4. Least square method.

Free Hand Curve Method

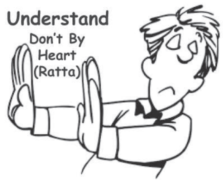
By freehand curve method, the trend is determined by just inspecting the plotted points on a graph sheet. Observe the up and down movements of the points. Smooth out the irregularities by drawing a freehand curve or line through the scatter points. The curve so drawn would give a general notion of the direction of the change. Such a freehand smoothed curve eliminates the short-time swings and shows the long period general tendency of the changes in the data.

Example: **Fit a trend line to the following data by the freehand method,**

Year	2001	2002	2003	2004	2005	2006	2007	2008	2009
Sales (million tonnes)	19	22	24	20	23	25	23	26	25

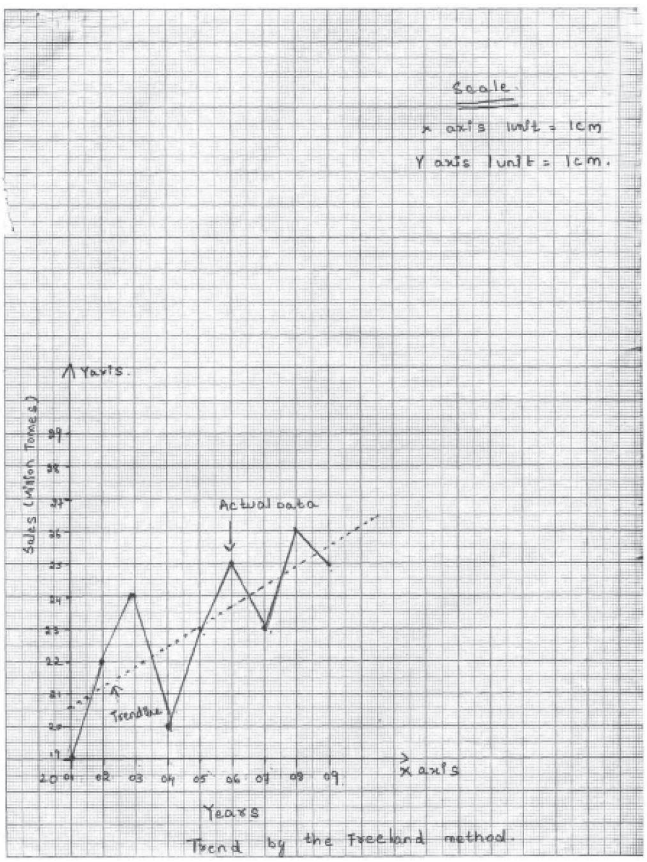
Solution :

Steps



1. Time series data is plotted on the graph
2. The direction of the trend is examined on the basis of the plotted data (dots)
3. A straight line is drawn which shows the direction of the trend.

The actual data and the trend line are shown in the following graph.



Drawing a smooth freehand curve requires a personal skill and judgement. The drawn curve should pass through the plotted points in such a manner that the variations in one direction are approximately equal to the variation in other direction. Different persons, however, draw different curves at different directions, with different slopes and in different styles. This may lead to different conclusions. To overcome these limitations, we can use the semi-average method of measuring the trend.

3.2.1 Semi Averages Methods

Q13. Explain how trend analysis is done by semi averages method. State merits and demerits of semi average method.

Answer :

Trend Analysis through Semi Average Method

The average between two time periods is referred as semi average. The procedure followed for semi-average method is as follows,

Step-1: The entire time series is classified into two equal parts with respect to time. For even period, equal split. For odd period, equal parts obtained by omitting middle period.

Step-2: Compute the arithmetic mean of time series values for each half separately. These means are called semi-averages.

Step-3: Semi averages are plotted as points against the middle point of the respective time periods covered by each part.

Step-4: The line joining these points gives the straight line trend fitting the given data.

Merits of Semi-Average Method

The following are some of the merits of semi-average method,

1. It does not depend on personal judgement.
2. It is easy to apply and understand.
3. By extending the line in both direction, we can get past and future estimates.

Demerits of Semi-Average Method

The following are some of the demerits of semi-average method,

1. It assumes the presence of linear trend which may not exist.
2. Usage of arithmetic mean for obtaining semi averages is questionable.
3. The values of trend are not precise and reliable.

Example

Using the following data, fit a trend line by using the method of semi-averages,

Year	1996	1997	1998	1999	2000	2001	2002
Output	700	900	1100	900	1300	1000	1600

Solution :

Step 1

The data provided in the problem is of seven years i.e., (an odd number). Thus, the middle year [1999] shall be ignored and the remaining years are divided into two equal time periods and their arithmetic averages is computed as follows,

$$\text{Average of the first three years} = \frac{700+900+1100}{3} = \frac{2700}{3} = 900$$

$$\text{Average of the last three years} = \frac{1300+1000+1600}{3} = \frac{3900}{3} = 1300$$

Therefore, the semi-averages are $\boxed{900}$ and $\boxed{1300}$.

Step 2

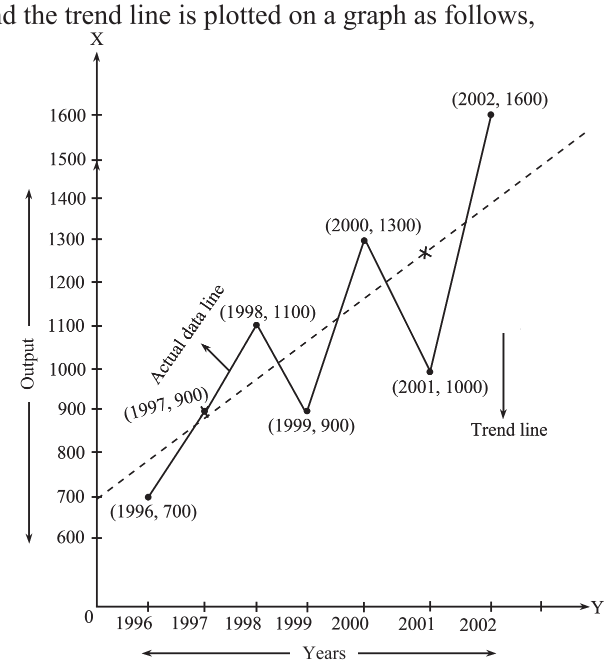
The next step is to plot the semi-averages against the mid-point (middle year) of each time period. Thus, it would be year 1997 and 2001 respectively.

Step 3

The plotted points are joined in order to derive the trend line using the semi average method.

Step 4

The original data and the trend line is plotted on a graph as follows,



3.2.2 Moving Averages Method

Q14. What is Moving Average Method? Discuss the method of moving averages in measuring trend. What are its merits and limitations?

Answer :

Moving Average Method

In moving average method, the average value for a number of years (month or weeks) is secured and this average is taken as the normal or trend value for the unit of time falling at the middle of the period covered in the calculation of the average. The effect of averaging is to give a smoother curve, lessening the influence of the fluctuations that pull the annual figures away from the general trend. The period of moving average is decided in the light of the length of the cycle. More applicable to data with cyclical movements.

Formula for 3 yearly moving average will be,

$$\frac{a+b+c}{3}, \frac{b+c+d}{3}, \frac{c+d+e}{3} \ldots$$

Formula for 5 yearly moving average will be,

$$\frac{a+b+c+d+e}{5}, \frac{b+c+d+e+f}{5}, \frac{c+d+e+f+g}{5} \ldots$$

Methods of Moving Average

The following two methods are followed in moving averages,

(i) Odd Yearly Method

Step-1: Calculate 3/5...yearly totals

Step-2: Now compute 3/5 yearly average by dividing the totals calculated in step-1 by the respective number of years. i.e., 3/5/...

Step-3: Short term oscillations are calculated using the formula,

$$Y - Y_e$$

Where, Y - Actual value and Y_e - Estimated value.

(ii) Even Yearly Method

Example: 4 years

Step-1: Calculate 4 yearly moving totals and place at the centre of middle two years of the four years considered.

Step-2: Divide 4 yearly moving totals by 4 to get 4 yearly average.

Step-3: Take a 2 period moving average of the moving average which gives the 4 yearly moving average centered.

Merits of Moving Average

The merits of moving average are as follows,

1. It is a simplest method among all mathematical methods of fitting trend.
2. It is a flexible method, even if a few more observations are to be added, the entire calculations are not changed.
3. The cyclical fluctuations are automatically eliminated when the period of moving average coincides with the period of the cycle.
4. Its curve is determined by the data rather than the statisticians choice of mathematical function.

Limitations of Moving Average

The following are the limitations of moving averages,

1. Its trend values cannot be computed for all the years. For example, in a 5 yearly moving we cannot compute trend values for the first two and the last two years.
2. It is difficult to decide the period of moving average since there is no hard and fast rule for the purpose.
3. It cannot be used in forecasting as it is not represented by any mathematical function.
4. When the trend is not linear, the moving average lies either above or below the true sweep of the data.

PROBLEMS ON MOVING AVERAGE METHOD

Q15. From the following data, calculate trend values using Four Yearly Moving Averages,

Year	2009	2010	2011	2012	2013	2014	2015	2016	2017
Production (in tons)	506	620	1036	673	588	696	1116	738	663

Solution : (Model Paper-I, Q11(b) | May/June-19, Q11(a) (OU) | Oct./Nov.-12, Q13(a) (OU))

Calculation of Trend by 4-yearly Moving Averages

Year (1)	Production (in Tonnes) (2)	4 yearly Moving Totals (3)	4 yearly Moving Averages (4) = (3) ÷ 4	2 Point Moving Totals (5)	4 yearly Moving Average Centered (6) = (5) ÷ 2
2009	506	–			
2010	620	–			
2011	1036	2835	708.75		
2012	673	2917	729.25	1438	719
2013	588	2993	748.25	1477.5	738.75
2014	248	3073	768.25	1516.50	758.25
2015	280	3138	784.5	1552.75	776.375
2016	300	3213	803.25	1587.75	793.875
2017	663	–			

Q16. Find the 4 yearly moving averages from the following data:

Year	2008	2009	2010	2011	2012	2013	2014	2015	2016	2017
Production (in Tonnes)	150	170	196	180	190	216	248	280	300	320

Solution : (Model Paper-III, Q11(a) | May/June-18, Q11(a) (OU))

Calculation of 4 yearly Moving Averages

Year (1)	Production (in Tonnes) (2)	4 yearly Moving Totals (3)	4 yearly Moving Averages (4) = (3) ÷ 4	2 Point Moving Totals (5)	4 yearly Moving Average Centered = (5) ÷ 2 (6)
2008	150	–	–	–	–
2009	170	–	–	–	–
		696	174		
2010	196			358	179
		736	184		
2011	180			379.5	189.75
		782	195.5		
2012	190			404	202
		834	208.5		
2013	216			442	221
		934	233.5		
2014	248			494.5	247.25
		1,044	261		
2015	280			548	274
		1,148	287		
2016	300	–			
2017	320	–			

Q17. From the following sales data, calculate the five year moving average.

Year	Sales (in ₹ ' 000)
2005	710
2006	705
2007	680
2008	687
2009	757
2010	629
2011	644
2012	783
2013	781
2014	805
2015	872

Solution : Oct./Nov.-16, Q13(a) (OU)

Calculation of 5-years Moving Averages

Year (1)	Annual Sales (in ₹ '000) (2)	5 Years moving Total (3)	5 Years moving average (4) = (3) ÷ 5
2005	710	–	–
2006	705	–	–
2007	680	3539	707.80
2008	687	3458	691.60
2009	757	3397	679.40
2010	629	3500	700.00
2011	644	3594	718.80
2012	783	3642	728.40
2013	781	3885	777.00
2014	805	–	–
2015	872	–	–

3.2.3 Least Square Method

Q18. What is the least square method? What are the merits and demerits of this method?

Model Paper-II, Q11(a)

OR

What is the least square method and explain its advantages and disadvantages?

Answer : May/June-19, Q8(b) (MGU)

Least Square Method

The least square method is a statistical procedure which is used to find the best fit curve for the set of data where different variables are involved. This method is mostly used for the time series of data in which the relationship of two or more variables is difficult to identify. Least square method provides a trend line of best firt in the form of curve in order to represents the relationship between a known and unknown variable. Because of this reason that trend line is also called as line of best fit. This line can be a straight line trend or parabolic trend through which sum of squares of the distance from different points is reduced or minimized.

The straight line trend equation is in the form of $Y_e = a + bX$

Where, Y denotes the trend value of the dependent variable

X denotes the independent variable.

a and b are constants.

The values of a and b are obtained by solving the following normal equations.

$$\Sigma Y = Na + b\,\Sigma X$$

$$\Sigma XY = a\,\Sigma X + b\,\Sigma X^2$$

Where, n represents the number of years in the series.

When $\Sigma X = 0$ the above normal equations are simplified to

$$a = \frac{\Sigma Y}{N}\ ;\ b = \frac{\Sigma XY}{\Sigma X^2}$$

By substituting a and b values in straight line trend equation $Y = a + bX$, we get the straight line equation which can be used for estimation of future values.

Merits of Least Squares

The following are the merits of least squares method,

1. It is a mathematical method of measuring trend and is free from subjectiveness.
2. It provides the line of best fit since it is this line from where the sum of positive and negative deviations is zero and the sum of square of deviations is the least.
3. It enables us to compute the trend values for all the given time periods in the series.
4. The trend equation can be used to estimate the values of the variable for any given time period 't' in future and the forecasted values are quite reliable.
5. It is the only technique which enables us to obtain the rate of growth per annum for yearly data in case of linear trend.

Demerits of Least Squares

The demerits of least squares are as follows,

1. It becomes necessary to do fresh calculation even if a single new observation is added.
2. Its calculations are quite tedious and time consuming as compared with other methods.
3. Its future predictions completely ignore the cyclical, seasonal and erratic fluctuations.
4. It cannot be used to fit growth curves, gomper t_z curve, logistic curve etc to which most of the business and economic time series conform.

PROBLEMS ON LEAST SQUARES METHOD

Q19. Below are given the figures of production (in thousand quintals) of a sugar factory.

Years	2001	2002	2003	2004	2005	2006	2007
Production	77	88	94	85	91	98	90

(i) Fit a straight line by the least squares method and tabulate the trend values.

(ii) What is the yearly increase in the production of sugar?

(Model Paper-III, Q11(b) | March/April-14, Q13(b) (OU))

OR

Fit a straight line by the Least Square Method and tabulate the trend values for the above data.

Year	2011	2012	2013	2014	2015	2016	2017
Production (in tons)	77	88	94	85	91	98	90

May/June-19, Q11(b) (OU)

Solution :

(i) Equation for straight line trend is $Y_e = a + bX$.

The value of a and b can attained by solving the following two normal equations,

$$\Sigma Y = Na + b\Sigma X \quad \text{... (1)}$$

$$\Sigma XY = a\Sigma X + b\Sigma X^2 \quad \text{... (2)}$$

Fitting of straight line trend by the method of least squares.

Year	Production Y	2004 (X) (Origin)	X^2	XY	Trend Value Y_c
2001	77	–3	9	–231	83
2002	88	–2	4	–176	85
2003	94	–1	1	–94	87
2004	85	0	0	0	89
2005	91	1	1	91	91
2006	98	2	4	196	93
2007	90	3	9	270	95
N = 7	$\Sigma Y = 623$	$\Sigma X = 0$	$\Sigma X^2 = 28$	$\Sigma XY = 56$	$\Sigma Y_C = 623$

$\Sigma Y_e = Na + b\Sigma X$

$623 = 7a + b(0)$

Since value of $\Sigma X = 0$, $\Sigma Y = Na$

$$a = \frac{\Sigma Y}{N} = \frac{623}{7} = 89$$

$$b = \frac{\Sigma XY}{\Sigma X^2} = \frac{56}{28} = 2$$

Hence, the value of $a = 89$ and $b = 2$

The equation of straight line trend is, $Y_e = 89 + 2X$

When $X = -3$, $Y = 89 + 2(-3) = 89 - 6 = 83$

When $X = -2$, $Y = 89 + 2(-2) = 89 - 4 = 85$

When $X = -1$, $Y = 89 + 2(-1) = 89 - 2 = 87$

When $X = 0$, $Y = 89$

When $X = 1$, $Y = 89 + 2(1) = 89 + 2 = 91$

When $X = 2$, $Y = 89 + 2(2) = 89 + 4 = 93$

When $X = 3$, $Y = 89 + 2(3) = 89 + 6 = 95$

(ii) The yearly increase in the production of sugar is $b = 2$ (in thousand quintals).

Q20. Production figure of a Textile Industry are as follows,

Year	2011	2012	2013	2014	2015	2016	2017
Production (in '000 units)	12	10	14	11	13	15	16

For the above data;

(i) Determine the straight line equation under the Least Square Method.

(ii) Find the Trend Values and show the trend line on a graph paper.

Solution : May/June-18, Q11(b) (OU)

(i) Determination of Straight Line Equation under the Least Square Method

The straight line trend equation is $Y_e = a + bX$

By solving these normal equations, we get a and b

Normal equations are,

$\Sigma Y = Na + b\Sigma X$

$\Sigma XY = a\Sigma X + b\Sigma X^2$

Fitting of Straight Line Trend

Year	Production (Y)	Deviations from 2014 (X)	X^2	XY
2011	12	– 3	9	–36
2012	10	– 2	4	–20
2013	14	–1	1	–14
2014	11	0	0	0
2015	13	1	1	13
2016	15	2	4	30
2017	16	3	9	48
N = 7	$\Sigma Y = 91$	$\Sigma X = 0$	$\Sigma X^2 = 28$	$\Sigma XY = 21$

Since $\Sigma X = 0$,

$$a = \frac{\Sigma Y}{N} = \frac{91}{7} = 13$$

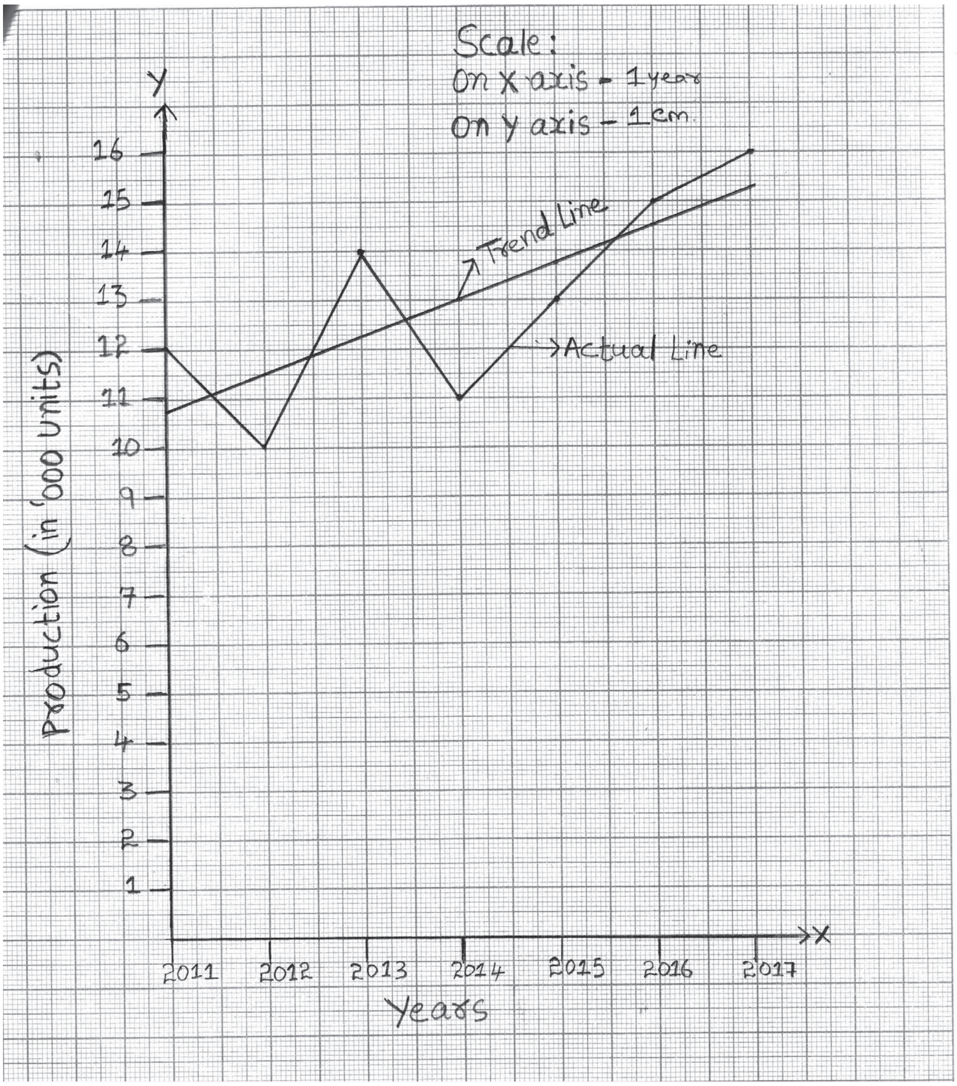
$$b = \frac{\Sigma XY}{\Sigma X^2} = \frac{21}{28} = 0.75$$

∴ The straight line trend $Y_e = a + bX$ is,

$Y_e = 13 + 0.75X$

(ii) Calculation of Trend Values

Year	X	Trend Y_e = 13 + 0.75X
2011	– 3	y = 13+ 0.75 (– 3) = 10.75
2012	– 2	y = 13+ 0.75 (– 2) = 11.5
2013	–1	y = 13+ 0.75 (– 1) = 12.25
2014	0	y = 13+ 0.75 (0) = 13
2015	1	y = 13+ 0.75 (1) = 13.75
2016	2	y = 13+ 0.75 (2) = 14.5
2017	3	y = 13+ 0.75 (3) = 15.25



Q21. Find the straight line tendency for the following data using least square method.

Year	2010	2011	2012	2013	2014	2015
Production	62	83	90	80	90	95

Solution : (Model Paper-II, Q11(b) | May/June-18, Q4(b) (KU))

Since the observations are even (N = 6), shift the origin to the middle of the two periods i.e., 2010 and 2013. Therefore, the values of X can be calculated as,

$$X = \frac{t - \left(\frac{2010 + 2013}{2}\right)}{\frac{1}{2}(\text{Interval})} \Rightarrow 2(t - 2012.5)$$

Solve Me with Concentration

$\therefore \quad X = 2(t - 2012.5)$

Fitting of Straight Line Trend by the Method of Least Squares

Year (t)	Production (Y)	X = 2 (t – 2012.5)	X^2	XY	Trend Value Y_e = 83.33 + 2.75 X
2010	62	– 5	25	– 310	69.58
2011	83	– 3	9	– 249	75.08
2012	90	– 1	1	– 90	80.58
2013	80	1	1	80	86.08
2014	90	3	3	270	91.58
2015	95	5	25	475	97.08
N = 6	ΣY = 500	ΣX = 0	ΣX^2 = 64	ΣXY = 176	Y_e = 499.98

$$a = \frac{\Sigma Y}{N} = \frac{500}{6} = 83.33$$

$$b = \frac{\Sigma XY}{\Sigma X^2} = \frac{176}{64} = 2.75$$

$\therefore$ The equation of straight line trend is,

$Y_e = 83.33 + 2.75X$

When $X = -5$, Y = 83.33 + 2.75 (– 5) = 83.33 – 13.75 = 69.58

When $X = -3$, Y = 83.33 + 2.75(– 3) = 83.33 – 8.25 = 75.08

When $X = -1$, Y = 83.33 + 2.75(– 1) = 83.33 – 2.75 = 80.58

When X = 1, Y = 83.33 + 2.75(1) = 83.33 + 2.75 = 86.08

When X = 3, Y = 83.33 +2.75(3) = 83.33 + 8.25 = 91.58

When X = 5, Y = 83.33 + 2.75(5) = 83.33 + 13.75 = 97.08.

Q22. From the following data calculate trend values based on least square method and estimate production in 1993.

Year	1982	1983	1984	1985	1986	1987	1988	1989
Production (000 tonnes)	58	56	55	51	47	38	35	32

Solution : May/June-19, Q8(a) (MGU)

Since the observations are even (N = 8), shift the origin to the middle of the two time periods i.e., 1985 and 1986. Therefore, the values of X can be calculated as,

$$X = \frac{t - \left(\frac{1985 + 1986}{2}\right)}{\frac{1}{2}(\text{interval})} \Rightarrow 2(t - 1985.5)$$

$\therefore \quad X = t - 1985.5$

Fitting of Trend Values by the Method of Least Squares

Year (t)	Production (000 tonnes) (Y)	X = 2 (t –1985.5)	XY	X^2	Trend Value Y_e = 46.5 + (– 2.04) X
1982	58	–7	–406	49	60.78
1983	56	–5	–280	25	56.7
1984	55	–3	–165	9	52.62
1985	51	–1	–51	1	48.54
1986	47	1	47	1	44.46
1987	38	3	114	9	40.38
1988	35	5	175	25	36.3
1989	32	7	224	49	32.22
N = 8	ΣY = 372	ΣX = 0	ΣXY = –342	ΣX^2 = 168	Y_e = 372

$$a = \frac{\Sigma Y}{N} = \frac{372}{8} = 46.5$$

$$b = \frac{\Sigma XY}{\Sigma X^2} = \frac{-342}{168} = -2.04$$

∴ The straight line trend is,

$Y_e = 46.5 + (-2.04)$ X

When $X = -7$, Y = 46.5 – 2.04 (– 7) = 46.5 + 14.28 = 60.78

When $X = -5$, Y = 46.5 – 2.04 (– 5) = 46.5 + 10.2 = 56.7

When $X = -3$, Y = 46.5 – 2.04 (– 3) = 46.5 + 6.12 = 52.62

When X = –1, Y = 46.5 – 2.04 (–1) = 46.5 + 2.04 = 48.54

When X = 1, Y = 46.5 – 2.04 (1) = 46.5 – 2.04 = 44.46

When X = 3, Y = 46.5 – 2.04(3) = 46.5 – 6.12 = 40.38

When X = 5, Y = 46.5 – 2.04(5) = 46.5 – 10.2 = 36.3

When X = 7, Y = 46.5 – 2.04(7) = 46.5 – 14.28 = 32.22.

Estimation of Production For the Year 1993

Deviation X for the year 1993 can be calculated as,

X = 2 (t – 1985.5)

= 2 (1993 – 1985.5)

= 3986 – 3971

= 15.

Y = 46.5 – 2.04 (15)

= 46.5 – 30.6

= 15.9

Thus, the production for the year 1993 is 15.9.

Q23. Fit a straight line trend by least squares method and estimate trend for 2015.

Year	2009	2010	2011	2012	2013	2014
Sales (₹ in ' 000s)	10	12	15	16	18	19

Solution : March/April-17, Q13(a) (OU)

The straight line trend is $Y_e = a + bX$

By solving these normal equations, we get a and b.

Normal equations are,

$\Sigma Y = Na + b\Sigma X$

$\Sigma XY = a\Sigma X + b\Sigma X^2$

Since, the even number of observation deviations are taken from both of the middle periods i.e., 2011 and 2012.

Year X	Sales (Y)	Deviations from Mid years (X)	X^2	XY
2009	10	$-2.5 \times 2 = -5$	25	–50
2010	12	$-1.5 \times 2 = -3$	9	–36
2011	15	$-0.5 \times 2 = -1$	1	–15
2012	16	$+0.5 \times 2 = 1$	1	16
2013	18	$+1.5 \times 2 = 3$	9	54
2014	19	$+2.5 \times 2 = 5$	25	95
N = 6	**$\Sigma Y = 90$**	**$\Sigma X = 0$**	**$\Sigma X^2 = 70$**	**$\Sigma XY = 64$**

Since, $\Sigma X = 0$, the normal equations simplify to,

$$a = \frac{\Sigma Y}{N} = \frac{90}{6} = 15$$

$$b = \frac{\Sigma XY}{\Sigma X^2} = \frac{64}{70} = 0.91$$

$$a + bx$$

$\therefore$ The straight line trend is $Y_e = 15 + 0.91X$.

Estimation of Sales For the Year 2015

For year 2015, deviation $X = 7$

$$\therefore \quad Y_e = a + bx$$

$$= 15 + 0.91(7) = 15 + 6.37$$

$$= 21.37$$

For the year 2015, the sales of the company is estimated to be ₹21.37.

Q24. The following data relate to the number of passenger cars (in millions) sold from 2004 to 2011.

Years	2004	2005	2006	2007	2008	2009	2010	2011
Number	6.7	5.3	4.3	6.1	5.6	7.9	5.8	6.1

(a) Fit a straight line trend to the data through 2009.

(b) Use your result in, (a) To estimate production in 2011 and compare with the actual production.

Solution : March/April-13, Q13(b) (OU)

Let, the equation of the straight line trend be $Y_e = a + b_X$, then,

$\Sigma Y = Na + b\Sigma X$

$\Sigma XY = a\Sigma X + b\Sigma X^2$

(a) Calculations to fit a straight line

Year	No. of Passenger cars (in millions) (Y)	Deviations from 2009 (X)	X^2	XY
2004	6.7	– 5	25	– 33.5
2005	5.3	– 4	16	– 21.2
2006	4.3	– 3	9	– 12.9
2007	6.1	– 2	4	– 12.2
2008	5.6	– 1	1	– 5.6
2009	7.9	0	0	0
2010	5.8	1	1	5.8
2011	6.1	2	4	12.2
N = 8	**ΣY = 47.8**	**ΣX = – 12**	**$\Sigma X^2 = 60$**	**$\Sigma XY = -67.4$**

$\Sigma Y = Na + b\Sigma X$ or $47.8 = 8a - 12b$ ... (1)

$\Sigma XY = a\Sigma X + b\Sigma X^2$ or $-67.4 = -12a + 60b$ ... (2)

Simplifying both the equations by multiplying equation(1) by 3 and equation (2) by 2, we get.

$$143.4 = 24a - 36b$$
$$-134.8 = -24a + 120b$$
$$8.6 = 84b$$

i.e., $b = \frac{8.6}{84} = 0.102$

Substituting the value of b in equation (1), we get,

$$47.8 = 8a - 12b$$
$$47.8 = 8a - 12(0.102)$$
$$47.8 = 8a - 1.224$$
$$8a = 49.024$$
$$a = \frac{49.024}{8} = 6.128$$

Hence, the required straight line equation is $Y_e = 6.128 + 0.102X$

(b) Estimating the sale of passenger cars in 2011 by substituting the value of $X = 2$.

$$Y_e = 6.128 + 0.102\ (2)$$
$$= 6.128 + 0.204$$
$$= 6.332$$

Thus, the estimated sale for 2011 is 6.332 million passenger cars whereas actual sales are 6.1 million passenger cars. There is some difference in actual figure and estimated figure because estimates depend on few assumptions.

Note

As number of passenger cars sold is given in question. So, estimated sales and actual sales are calculated instead of estimated production and actual production.

3.3 DESEASONALISATION OF DATA

Q25. What do you mean by deseasonalization of data? Explain it with an example problem.

Answer :

Deseasonalisation of Data

Deseasonalisation of data refers to the process of eliminating seasonal fluctuations from the given time series for, it facilitates in adjusting the given time series for seasonal fluctuations and therefore left out with variables like trend component, cyclical and irregular variations.

In multiplicative model of time series deseasonalised values are calculated by dividing the given values with the respective indices of seasonal variations.

$$\text{Deseasonalised data} = \frac{Y}{S} = \frac{TCSI}{S} = TCI$$

In additive model of time series, deseasonalised values are calculated by deducting seasonal variations from the given values,

$$\text{Deseasonalised data} = Y - S = (T+S+C+I) - S = T+C+I$$

Example

'XYZ' Co. estimates its average sales in a particular year to be ₹ 2,00,000. The seasonal indices of the sales data are as follows.

Month	Jan.	Feb.	March	April	May	June	July	Aug.	Sept.	Oct.	Nov.	Dec.
Seasonal	95	60	100	98	106	110	120	136	125	96	75	79

Using this information calculate monthly sales for the company. Assuming that there is no trend.

Solution :

$$\text{Seasonal effect} = \frac{\text{Seasonal index}}{100}$$

Average annual sale = ₹ 2,00,000

∴ Average monthly sales = 2,00,000 × Seasonal effect

Month (1)	Seasonal Indices (2)	Seasonal Effect $(3) = \frac{(2)}{100}$	Monthly Sales (4) = (3) × 2,00,000
January	95	[95/100] 0.95	1,90,000
February	60	[60/100] = 0.6	1,20,000
March	100	[100/100] = 1	2,00,000
April	98	[98/100] = 0.98	1,96,000
May	106	[106/100] = 1.06	2,12,000
June	110	[110/100] = 1.1	2,20,000
July	120	[120/100] = 1.2	2,40,000
August	136	[136/100] = 1.36	2,72,000
September	125	[125/100] = 1.25	2,50000
October	96	[96/100] = 0.96	1,92,000
November	86	[75/100] = 0.75	1,50,000
December	79	[79/100] = 0.79	1,58,000
	1200	**12**	**24,00,000**

3.4 USES AND LIMITATIONS OF TIME SERIES

Q26. State the uses and limitations of time series.

Answer :


Keep an eye on me

Uses/Utility of Time Series

The analysis of time series is not only used by the economist and business but it is also followed and used by the scientist, astronomist, geologist, sociologist, biologist and researchers. The uses or utility of time series analysis is as follows,

1. It Helps in Understanding Past Behaviour

It helps in understanding the past behaviour by considering the changes that have taken place in the past. With the help of past data they predict the future behaviour.

2. It Helps in Planning Future Operations

It helps in future planning by forecasting the events and their relationship. If regular occurrence of any event is there over a long period then such event is considered to predict the future.

3. It Helps in Evaluating Current Accomplishments

It helps in evaluating the performance by comparing the actual performance with the expected performance and analyse the reasons of variations if any. For example, If expected sale of a refrigerator is 10,000 for 2011-12 but actual sale is only 9000. By using time series analysis they can evaluate the reason for it's shortfall.

4. It Facilitates Comparison

The time series for different periods are compared and various conclusions are drawn upon it. But it is not necessary that everyone should believe on it. As statisticians are not foretellers, they cannot predict 100% accurate results for future events.

The future prediction could be possible only if they include the influences of various forces which will effect the series such as climate, customs and traditions and other factors like growth and declining factors and complex forces which will effect the production of business cycles and such analysis is examined carefully for number of times.

Limitations of Time Series

The limitations of time series are as follows,

1. It may not be used/applied in all the situations due to insufficient data.
2. It takes into consideration the environmental factors which result in variations.
3. It may result into errors or deviations, which need to be monitored.
4. It is difficult to predict cyclical and random variations.
5. When cyclical and random variations are ignored, forecasts based on the extension of the trend line and seasonal indices may lead to inaccuracy in some cases.

EXERCISE AND PRACTICE QUESTIONS

SHORT QUESTIONS

THEORY

Q1. What is Time Series? [Refer, Q1]

Q2. Utility of Time Series Analysis. [Refer, Q2]

May/June-19, Q4 (OU)

OR

What are the uses of time series? May/June-18, Q4 (OU)

Q3. Objectives of Time Series [Refer, Q3] May/June-19, Q3 (MGU)

Q4. What is seasonal variations? Write three features of seasonal variations. [Refer, Q4]

May/June-18, Q1(c) (KU)

Q5. What are the Components of Time Series? [Refer, Q5]

PROBLEMS

Q6. Fit a trend line to the following data by the Freehand Method. [Refer Similar, Q6]

Year	2001	2002	2003	2004	2005	2006	2007	2008	2009
Sales (₹)	20	23	25	21	24	26	24	27	26

Q7. Annual trend of Milk Consumption (Y) is 20.6 + 1.9 X. Convert the equation into monthly basis. [Refer Similar, Q7]

(Ans: Y_e = 1.717 + 0.0132X)

Q8. Calculate 3 yearly moving averages from the following data, [Refer Similar, Q8]

Year	2013	2014	2015	2016	2017	2018	2019
Sales (in million ₹)	82	88	90	92	94	96	98

(Ans: 2014 = 86.66 ; 2015 = 90 ; 2016 = 92 ; 2017 = 94 ; 2018 = 96)

Q9. Given the following equation Y_e = 420 + 3X. Time origin is 2006. Time unit is one year, shift the origin to 2011. [Refer Similar, Q9]

(Ans: Y_e = 427.5 + 3X)

Q10. Calculate five yearly moving averages for the following data: [Refer Similar, Q8]

Year	1981	1982	1983	1984	1985	1986	1987	1988	1989	1990
No. of students	332	317	357	392	402	405	410	417	405	431

March/April-16, Q8(OU)

(Ans: 5-yearly M.A's for 1983 to 1988 respectively are: 360, 374.6, 393.2, 405.2, 407.8, 413.6).

Q11. Given Y = 300 + 24 X (origin 2001) X-unit = 1 year. Y-unit = Annual profits (₹1000). Convert into monthly trend equation. [Refer Similar, Q7] March/April-11, Q8 (OU)

(Ans: Y_e= 25 + 0.1667X)

Q12. Given Y = 150 + 12X (origin 2009) X-unit = 1 year, Y-unit = annual profits ('000 ₹). Convert into monthly trend equation. [Refer Similar, Q7] Sept./Oct.-15, Q8 (OU)

(Ans: Y_e = 12.5 + 0.08333X)

ESSAY QUESTIONS

THEORY

Q13. What is Time Series? Explain the components of time series. [Refer, Q10]

Q14. Write the features of tendency. [Refer, Q11] May/June-18, Q4(a) (KU)

Q15. State the uses and limitations of time series. [Refer, Q26]

PROBLEMS

Q16. Compute the trend values by the method of semi-averages from the data given below, [Refer Similar, Q13]

Year	1992	1993	1994	1995	1996	1997	1998	1999
No. of sheep (in lakhs)	56	55	51	47	42	38	35	32

(Ans: Trend values (in lakhs) for the years 1992 to 1999 are: 59, 56, 50.5, 46.5, 41.5, 37, 32.5, 28).

Q17. The sale of a commodity in tonnes varied from January 1999 to December 1999 in the following manner:

280	300	280	280	270	240
230	230	220	200	210	200

Find a trend by the method of semi-average.

Fit a trend line from the following data by using semi-average method: [Refer Similar, Q13]

Year	1993	1994	1995	1996	1997	1998
Profits (in '000)	100	120	140	150	130	200

(Ans: Joining the points (1994, 120) and (1997, 160), we get the trend line).

Q18. Calculate 5 yearly moving averages from the following data. [Refer Similar, Q17]

Year	2002	2003	2004	2005	2006	2007
Production	100	105	115	90	95	85
Year	2008	2009	2010	2011	2012	2013
Production	80	65	75	70	75	80

March/April-15, Q13(a) (OU)

(Ans: 5-yearly moving averages are 101, 98, 93, 83, 80, 75, 73, 73).

Q19. Draw a trend line by the method of semi-averages from the following data,

Year	1984	1985	1986	1987	1988	1989	1990	1991
Sales '000 (unit)	200	120	128	192	204	126	224	228

Also predict the sales from the year 1993 from the graph. [Refer Similar, Q13]

(Ans: Sales for 1993 = 230).

Q20. Fit a straight line trends to the following data and estimate the likely profit for the year 2004. [Refer Similar, Q19]

Year	1995	1996	1997	1998	1999	2000	2001
Profits in lakhs rupee	60	72	75	65	80	85	95

March/April-16, Q13(b) (OU)

(Ans: Y_e = 76 + 4.857X Likely profit for the year 2004 is ₹ 105.142 lakhs)

Q21. Fit straight line trend to the following data by using least square method. [Refer Similar, Q19]

Year	2006	2007	2008	2009	2010	2011	2012
Production (in lakhs of tons)	48	50	58	52	45	41	49

Sept./Oct.-15, Q13(a) (OU)

(Ans: Y_e = 49 + (–1)X)

Q22. Fit a straight line trend by the method least squares to the following data and also predict the earnings for the year 2013, [Refer Similar, Q22]

Year	2005	2006	2007	2008	2009	2010	2011	2012
Earnings (₹ in lakhs)	38	40	65	72	69	60	87	95

Oct./Nov.-14, Q13(a) (OU)

(Ans: Y_e = 65.75 + 3.66 X, Predicted earnings for the year 2013 is 98.69 lakhs).

INTERNAL ASSESSMENT/EXAM

I Multiple Choice

1. __________ helps in evaluating current accomplishments. [ ]
   (a) Correlation (b) Time series
   (c) Trend analysis (d) Index numbers

2. _________ refers to only smooth, regular, long-term movement of the data. [ ]
   (a) Trend (b) Random variation
   (c) Cyclical variation (d) None of the above

3. Deaths due to epidemics, bank interest rate, inflation rate etc are the examples of __________. [ ]
   (a) Upward trend (b) Downward trend
   (c) Both (a) and (b) (d) All the above

4. Which of the following are the types of periodic movements? [ ]
   (a) Seasonal fluctuations (b) Cyclical fluctuations
   (c) Both (a) and (b) (d) Secular fluctuations

5. Methods which can be used for measuring trend component of time series are __________. [ ]
   (a) Semi averages (b) Moving averages
   (c) Least square (d) All the above

6. Short-term oscillations are calculated by using the __________ formula. [ ]
   (a) $Y + Y_e$ (b) $Y \div Y_e$
   (c) $Y - Y_e$ (d) $Y \times Y_e$

7. The straight line trend equation is in the form of __________ [ ]
   (a) $X = a + bX$ (b) $Y = a + bX$
   (c) $X = a - bX$ (d) $Y = a - bX$

8. ____________ method is used to fit either a straight line trend or a parabolic trend. [ ]
   (a) Least square (b) Semi averages
   (c) Freehand curve (d) Moving average

9. Seasonal Effect = __________ [ ]
   (a) $\frac{\text{Seasonal Index}}{50}$ (b) $\frac{\text{Seasonal Index}}{10}$
   (c) $\frac{\text{Seasonal Index}}{30}$ (d) $\frac{\text{Seasonal Index}}{100}$

10. In multiplicative model of time series deseasonalised values are calculated by __________ the given values with the respective indices of seasonal variations. [ ]
    (a) Dividing (b) Multiplying
    (c) Adding (d) Deducting

II Fill in the Blanks

1. __________ is an arrangement of statistical data in a chronological order i.e., in accordance with its time of occurrence.
2. __________ refers to the general tendency of the time series data to increase or decrease over a long period of time.
3. The wave like movements in a time series with period of oscillation more than one year are called __________.
4. Wars, earthquakes, floods, fires, strikes, lockouts etc., are the examples of __________.
5. Under __________ method, the average value for a number of years is secured and this average is taken as the normal or trend value.
6. __________ is the most widely used method and provides us with a mathematical device to obtain an objective fit to the trend of a given time series.
7. $\Sigma XY =$ __________.
8. __________ refers to the process of eliminating seasonal fluctuations from the given time series.
9. In addictive model of time series, deseasonalised values = __________.
10. __________ are plotted as points against the middle point of the respective time periods covered by each part.

KEY

I. Multiple Choice

1. (b)
2. (a)
3. (b)
4. (c)
5. (d)
6. (c)
7. (b)
8. (a)
9. (d)
10. (a)

II. Fill in the Blanks

1. Time series
2. Secular trend
3. Cyclical fluctuations
4. Random fluctuations
5. Moving average
6. Least squares
7. $a\Sigma X + b\Sigma X^2$
8. Deseasonalisation of data
9. $Y - S = (T + S + C + I) - S = T + C + I$
10. Semi averages.

III Very Short Questions and Answers

Q1. Define Time Series.

Answer :

Time series is an arrangement of statistical data in a chronological order i.e., accordance with its time of occurrence.

Q2. What is Secular Trend?

Answer :

Secular trend refers to the general tendency of the time series data to increase or decrease over a long period of time.

Q3. Write a note on method of Least Squares.

Answer :

Least squares is the most widely used method and provides us with a mathematical device to obtain an objective fit to the trend of a given time series.

Q4. What do you mean by Deseasonalisation of Data?

Answer :

Deseasonalisation of data refers to the process of eliminating seasonal fluctuations from the given time series for, it facilitates in adjusting the given time series for seasonal fluctuations and out with variables like trend component, cyclical and irregular variations.

Q5. Write about Seasonal Fluctuations.

Answer :

Seasonal fluctuations are those periodic movements which occur regularly every year and have their origin in the nature of the year itself.

UNIT 4

Probability

SYLLABUS

Probability – Meaning - Experiment – Event - Mutually Exclusive Events - Collectively Exhaustive Events - Independent Events - Simple and Compound Events - Basics of Set Theory – Permutation – Combination - Approaches to Probability: Classical – Empirical – Subjective - Axiomatic - Theorems of Probability: Addition – Multiplication - Baye's Theorem.

LEARNING OBJECTIVES

- ✓ *Concept of Probability with its Meaning and Importance.*
- ✓ *Basic Concepts of Probability – Experiment, Event, Mutually Exclusive Events, Collectively Exhaustive Events, Independent Events, Simple and Compound Events.*
- ✓ *Basics of Set Theory.*
- ✓ *Concept of Permutation and Combination.*
- ✓ *Approaches to Probability – Classical, Empirical, Subjective and Axiomatic*
- ✓ *Theorems of Probability – Addition and Multiplication Theorems*
- ✓ *Baye's Theorem and its Applications.*

INTRODUCTION

Probability can be defined as the chance or 'likelihood of occurrence' of an experiment or event. Probability of any event ranges from 0 to 1.

A probability is a quantitative measure of uncertain events. It helps in determining the chances of the occurrence of an event. The systematic and extensive study of probability theory was made by 'B.Pascal', 'Pierre de Fermat', Jacques Bernoulli' in mid-seventeeth century.

The collection of finite or infinite number of objects with some common property is called Set. The objects belonging to the set are called Members or Elements of the set.

Permutations refer to different arrangements of objects in a set wherein all the elements are distinguishable. In these arrangements, an individual object is not repeated. If some objects are similar, then the permutations will be affected.

A combination is a selection of objects irrespective of their arrangement. The number of combinations of objects in different ways is entirely different from the number of their permutations.

Based on the concept of probability there are four different approaches of probability, which are explained below, Classical or priori approach, Relative frequency approach, Subjective approach, Axiomatic approach.

PART-A SHORT QUESTIONS AND ANSWERS

Q1. Define Probability.

Answer : Model Paper-III, Q6


I am Simple and Easy

Probability can be defined as the chance or 'likelihood of occurrence' of an experiment or event. Probability of any event ranges from 0 to 1. The term 'probability' was coined by an Italian mathematician, 'Galileo'. He was the first man who measured probability quantitatively while dealing with the problems associated with rolling of dice in gambling.

The word 'probability' or 'chance' is the most common word used in our day-to-day life. For example, in our daily life we use certain statements like, Probably he may win the elections etc.

A probability is a quantitative measure of uncertain events. It helps in determining the chances of the occurrence of an event. The systematic and extensive study of probability theory was made by 'B.Pascal', 'Pierre de Fermat', Jacques Bernoulli' in mid-seventeeth century.

Q2. What are mutually exclusive events, non-mutually exclusive events and dependent events?

Model Paper-I, Q5

OR

Explain (i) Mutually exclusive events and (ii) Not-mutually exclusive events.

(Refer Only Topics: Mutually Exclusive Events, Non-Mutually Exclusive Events)

May/June-18, Q5 (OU)

OR

Explain:

(i) Mutually exclusive events and

(ii) Dependent events.

(Refer Only Topics: Mutually Exclusive Events, Dependent Events)

Answer : May/June-19, Q6 (OU)

Mutually Exclusive Events

Two events are said to be mutually exclusive (or incompatible) when both cannot happen simultaneously in a single trial. For example, if a coin is tossed, either a head or a tail appears but both cannot occur simultaneously.

Non-Mutually Exclusive Events

When two events can occur simultaneously in a single trial then such events are said to be non-mutually exclusive events. For example, from a pack of cards, drawing a red card and drawing a queen are the two events. These two events can occur simultaneously while drawing a red queen. Hence, these two events are said to be non-mutually exclusive events which can occur at the same time.

Dependent Events

Dependent events are the events in which the occurrence or non-occurrence of one event in any one trial influences the probability of occurrence of other events in other trials. For example, When a coin is tossed, getting a head or a tail are mutually exclusive and collectively exhaustive. Hence, if we get tail then head is considered as its complementary event.

Q3. What do you mean by Conditional Probability?

Answer :

Two events 'A' and 'B' are said to be dependent events when 'B' can occur only when 'A' is known to have occurred or vice versa. Such a probability is called as 'Conditional Probability' for dependent events and is denoted by $P(A/B)$.

For statistically independent events, the conditional probability is same as that of the marginal probability.

For independent events,

$$P(A/B) = \frac{P(A \cap B)}{P(B)}$$

For dependent events, $P(A/B) = P(A)$.

Q4. What is Joint Probability?

Answer :

A joint probability is the probability of occurrence of two or more simple events. It is the product of two marginal probabilities.

Joint probability of A and B is represented as $P(A \cap B)$

$\therefore \quad P(A \cap B) = P(A).P(B)$

Example

Probability of red queen picked from a pack of cards. It consists of two simple events of picking a red card and a queen.

Q5. What is Marginal Probability?

Answer :

Marginal probability is the simple probability of the occurrence of an event. It is also known as 'unconditional' probability or 'single probability'.

For example, when a single unbiased coin is tossed, the probability that it is a head is $P(H) = \frac{1}{2}$ and probability that it is a tail is $P(T) = \frac{1}{2}$.

These two events are independent and do not overlap one another i.e., the events are statistically independent of the outcomes of next coin been tossed.

The individual probabilities obtained in this case i.e., $P(H)$ or $P(T)$ are called as marginal probabilities.

Q6. When two dice are thrown, find the probability that the sum of the numbers is either 10 or 11.

Answer : (Model Paper-I, Q6 | May/June-18, Q6 (OU))

When two dice are thrown, the sample space, S contains $6^2 = 36$ points as shown below:

$$\begin{aligned} S = \{&(1, 1), (1, 2), (1, 3), (1, 4), (1, 5), (1, 6),\\ &(2, 1), (2, 2), (2, 3), (2, 4), (2, 5), (2, 6),\\ &(3, 1), (3, 2), (3, 3), (3, 4), (3, 5), (3, 6),\\ &(4, 1), (4, 2), (4, 3), (4, 4), (4, 5), (4, 6),\\ &(5, 1), (5, 2), (5, 3), (5, 4), (5, 5), (5, 6),\\ &(6, 1), (6, 2), (6, 3), (6, 4), (6, 5), (6, 6)\} \end{aligned}$$

The number of possible outcomes for getting the sums of 10 or 11 is,

(4, 6) (5, 5) (5, 6) (6, 4) (6, 5)

$\therefore$ The required probability is $\frac{5}{36}$.

Q7. One card is drawn at random from a pack of 52 cards. What is the probability that it is either a king or a queen?

Answer : May/June-19, Q7 (OU)

Let 'A' be the event of drawing a king from a standard pack of 52.

'B' be the event of drawing a queen.

$P(A)$ = Probability of drawing a king

$P(B)$ = Probability of drawing a queen.

As, the probability of drawing either a king or a queen has to be determined by drawing a single card, the events are said to be mutually exclusive.

For mutually exclusive events,

$$\boxed{P(A \cup B) = P(A) + P(B)}$$

(By addition theorem of probability)

Given data,

$P(A) = \frac{4}{52}$ (As there are 4 kings in a standard pack of 52)

$P(B) = \frac{4}{52}$ (As there are 4 queens in a standard pack of 52)

$$\therefore P(A \cup B) = \frac{4}{52} + \frac{4}{52} = \frac{8}{52} = \frac{2}{13}$$

Therefore, the probability of drawing either a king or a queen $= \frac{2}{13}$.

Q8. n(A) = 35, n(B) = 30, n(A ∩ B) = 20 then find ∩ (A ∪ B).

Answer : **(Model Paper-II, Q6 | May/June-18, Q1(d) (KU))**

Given that,

$n(A) = 35$

$n(B) = 30$

$n(A \cap B) = 20$

$n(A \cup B) = n(A) + n(B) - n(A \cap B)$

$= 35 + 30 - 20$

$\therefore \quad n(A \cup B) = 45$

Q9. How many 3 letter words can be formed from the English world "SUCCESS"?

Answer : **(Model Paper-III, Q7 | May/June-18, Q1(e) (KU))**

The English word success formed by seven letters, which consists of,

$S = 3$ times

$C = 2$ times

$U = 1$ time and

$E = 1$ time

Number of ways for arranging these letters $= \frac{7!}{3!} = 840$ ways

Therefore, in the word "SUCCESS" 'S' is the letter which is repeating three times.

Q10. Find the value of $6P_4$, $5P_2$.

Answer : **May/June-18, Q1(f) (KU)**

(i) $6P_4$

$$nP_r = \frac{n!}{(n-r)!}$$

Here, $n = 6$

$r = 4$

$$6P_4 = \frac{6!}{(6-4)!}$$

$$= \frac{6!}{2!}$$

$$= \frac{6\times5\times4\times3\times2\times1}{2\times1}$$

$$= 6 \times 5 \times 4 \times 3$$

$\therefore$ Value of $6P_4 = 360$

(ii) $5P_2$

$$nP_r = \frac{n!}{(n-r)!}$$

Here,

$n = 5$

$r = 2$

$$5P_2 = \frac{5!}{(5-2)!}$$

$$= \frac{5!}{3!}$$

$$= \frac{5\times4\times3\times2\times1}{3\times2\times1}$$

$$= 5 \times 4$$

$\therefore$ Value of $5P_2 = 20$

Q11. Calculate probability of 53 Mondays in a leap year.

Answer : (Model Paper-II, Q7 | May/June-19, Q4 (MGU))

A leap year consists of 366 days

A year has 52 weeks. Hence, there will be 52 mondays for sure.

52 weeks = 52 × 7 = 364 days

366 – 364 = 2 days

In a leap year, there will be 52 Mondays and 2 days will be left.

This 2 days may take the following combinations,

(Sunday, Monday), (Monday, Tuesday), (Tuesday, Wednesday), (Wednesday, Thursday), (Thursday, Friday), (Friday, Saturday), (Saturday, Sunday).

Out of 7 outcomes, favorable outcomes are 2.

Hence, the probability of getting 53 Mondays in a leap year is 2/7.

$\therefore$ It means that, there is a increase in prices over base level by 130.537%.

PART-B ESSAY QUESTIONS AND ANSWERS

4.1 PROBABILITY – MEANING AND IMPORTANCE

Q12. What do you mean by probability? Explain the importance of probability.

Answer : Model Paper-II, Q12(a)

Probability

Probability can be defined as the chance or 'likelihood of occurrence' of an experiment or event. Probability of any event ranges from 0 to 1. The term 'probability' was coined by an Italian Mathematician, 'Galileo'. He was the first man who measured probability quantitatively while dealing with the problems associated with rolling of dice in gambling.

The word 'probability' or 'chance' is the most common word used in our day-to-day life. For example, in our daily life we use certain statements like 'Probably he may win the elections", "It is likely that India may win the match", "She may score above 90% in the upcoming examination" etc.

A probability is a quantitative measure of uncertain events. It helps in determining the chances of the occurrence of an event. The systematic and extensive study of probability theory was made by 'B.Pascal', 'Pierre de Fermat', Jacques Bernoulli' in mid-seventeeth century.

Importance of Probability

The various practical applications or importance of the theory of probability are as follows,

1. It helps in taking effective decisions under uncertain conditions.
2. It reduce complications in betting and games.
3. It is useful in economic decision making. It is very helpful in situations of risk and uncertainty.
4. It helps in carrying out different types of scientific investigations.
5. The fundamental laws of statistics, viz the law of statistical regularity and law of inertia of large numbers depends mainly on theory of probability.
6. Decision theories are based on fundamental laws of probability and expected value.
7. The empirical probability concept, based on experimental tests, provides scope for the application of probability to real life situations.
8. Subjective probabilities are used when it is not possible to do actual measurement. This has added a new dimension to the theory of probability. Such probabilities are revised by the application of Baye's rule.
9. Probability models helps in making predictions.
10. Probability tree diagrams helps in calculating probability values of different situations.

4.1.1 Experiment, Event, Mutually Exclusive Events, Collectively Exhaustive Events, Independent Events, Simple and Compound Events

Q13. What are the key concepts of probability?

Answer : (Model Paper-I, Q12(a) | May/June-18, Q5(a) (KU))

The key concepts or terms of probability are as follows,

1. Experiment

An experiment is also referred as random experiment. It is a process or activity which leads to a particular outcome of several possible outcomes. The outcome which is going to be derived through random experiment is not known until its occurence (i.e., the outcome of random experiment is not predictable). But, the number of possible outcomes can be known. There may be fixed or infinite number of outcomes for a particular experiment. Outcome from random experiment may be numerical or non-numerical in nature.

Examples

Some of the examples of random experiment are as follows,

(i) Drawing a card from a group of shuffled cards.

(ii) Finding the chance of getting 6 when a dice is thrown.

2. Outcome

The result of a random experiment is usually referred as an outcome.

Example

If we toss a coin, the outcome may be a head or a tail. In such a case, number of outcomes = 2.

3. Event

An event is a possible outcome of an experiment or a result of trial. Basically there are two types of events. Simple and compound event.

(i) Simple Event

The probability of happening or non-happening of a single event is considered as a simple event.

Example

When we are selecting two black coins from a box containing 10 white and 5 black coins.

(ii) Compound Event

If the joint occurrence of two or more event is considered then it is known as 'compound event' or 'composite event'.

Example

A box containing 5 white balls, 3 black balls and 8 red balls, when we draw 2 white balls in first draw, 2 black balls in second draw and 5 red balls in third draw.

4. Mutually Exclusive Events

When two events cannot occur simultaneously in a single trial then such events are called as mutually exclusive or incompatible events.

Examples

(i) When a single coin is tossed, either a head or a tail can turn up, both cannot come at the same time.

(ii) At a single point of time, a person can be alive or dead.

5. Non-mutually Exclusive Events

When two events can occur simultaneously in a single trial then such events are said to be non-mutually exclusive events.

Example

From a pack of cards, drawing a red card and drawing a queen are the two events. These two events can occur simultaneously while drawing a red queen. Hence, these two events are said to be non-mutually exclusive events which can occur at the same time.

6. Collectively Exhaustive Events

Collectively exhaustive events are those events whose totality contains all the potential outcomes of a random experiment.

Example

When a dice is thrown, the total possible outcomes are 1, 2, 3, 4, 5 and 6 and thus the number of exhaustive cases is 6.

7. Equally Likely Events

Events are considered as equally likely events when the probability of occurrence of all the events is equal. These events are also called as 'equally, probable events'.

Example

If a coin is tossed, the two possible outcomes are head and tail. The probability of their occurrence is equal (i.e, $\frac{1}{2}$)

8. Independent Events

Two or more events are considered as independent events, when the outcome of one event does not influences and is not influenced by the other event.

Example

When a student has appeared in physics and chemistry examinations, his marks obtained in physics is independent of the marks obtained in chemistry.

9. Dependent Events

Dependent events are the events in which the occurrence or non-occurrence of one event in any one trial influences the probability of occurrence of other events in other trials.

Example

When a card is drawn from a pack of playing cards and is not replaced then this changes the probability of occurrence of the second card. Here, probability of occurrence of second event is dependent on the occurrence of first event.

10. Complementary Events

Two events are said to be complementary events if they are mutually exclusive and collectively exhaustive.

Example

When a coin is tossed, getting a head or a tail are mutually exclusive and collectively exhaustive. Hence, if we get tail then head is considered as its complementary event.

11. Sample Space

Sample space refers to the set of all possible outcomes for a particular random experiment. It is denoted by the capital letter S.

$$S = \{E_1, E_2, E_3.........E_n\}$$

All the possible outcomes (sample space) can also be denoted by a tree diagram.

The main property of sample space is that, only one outcome from sample space can occur at a time (i.e.,) two or more outcome cannot occur simultaneously.

Example

In a experiment of tossing two coins, following sample events are the possible outcomes,

E_1 = HH, E_2 = HT, E_3 = TH, E_4 = TT

Here sample space, $S = \{E_1, E_2, E_3, E_4\}$

4.2 BASICS OF SET THEORY

Q14. Define set. How is it denoted and what are the two different ways of representing a set along with an example?

Answer :

Set

The collection of finite or infinite number of objects with some common property is called Set. The objects belonging to the set are called Members or Elements of the set.

Examples

(i) A group of people

(ii) A collection of stamps

(iii) A pair of glasses.

Notations

A set is usually denoted by capital letters with or without subscripts. Lower case letters are used to signify the elements of the set.

Representation of Set

There are two ways for representing a set. They are as follows,

1. Roaster Method

In Roaster Method, the elements are enclosed within the "{ }" brackets.

Example

Set of natural numbers

$N = \{1, 2, 3, 4, ...\}$

2. Set-builder Form

In Set-builder form method, the elements are described with respect to their common property.

Example

$\{x/x$ is a natural number$\}$

Note

If an element x belongs to set A, then it is represented as $x \in A$.

If an element x does not belong to set A, then it is represented as $x \notin A$.

Q15. Define the following with suitable example:

(i) Finite and infinite sets

(ii) Inclusion sets (Subset)

(iii) Equality of sets (Equal sets).

(iv) Proper subset

(v) Universal set

(vi) Null set

(vii) Singleton set

(viii) Power set

(ix) Disjoint set.

Answer :

(i) Finite and Infinite Sets

If a set contain finite (i.e., countable) number of elements, then that set is referred to as finite set. On the other hand, if a set contains infinite (i.e., uncountable) number of elements, then that set is referred to as infinite set.

Example

{2, 4, 6, 8, ...} finite set

{1, 3, 5, 7, 9, ...} infinite set

Note

The order of finite set is denoted by $n(A)$ and the order of infinite set is ∞.

(ii) Inclusion Sets (Subset)

If X and Y are any two sets such that every element of X belongs to the set Y, then X is included in Y and X is called as subset of Y which is denoted by $X \subseteq Y$ or $Y \subseteq X$.

Example

$X_1 = \{1, 5, 7, 9\}$

$X_2 = \{1, 7\}$

X_2 is included in X_1

$X_2 \subseteq X_1$

Properties of Set Inclusion

For any three sets X, Y and Z

(a) Reflexive: $X \subseteq X$

(b) Transitive: $(X \subseteq Y)$ and $(Y \subseteq Z)$ then $X \subseteq Z$.

Note

Symmetric property is not satisfied (i.e., if $X \subseteq Y$, then it is not necessary that $Y \subseteq X$). Symmetric property is satisfied only when the sets are equal.

(iii) Equality of Sets (Equal Sets)

Two sets X and Y are said to be equal if and only if $X \subseteq Y$ and $Y \subseteq X$.

i.e., $X = Y \quad \Leftrightarrow (X \subseteq Y \wedge Y \subseteq X)$

Example

(a) Let $X = \{1, 2, 3, 4, 5\}$

$Y = \{1, 3, 5, 2, 4\}$

Here, $X \subseteq Y$ and $Y \subseteq X$ which imply $X = Y$.

(b) Let $X = \{\{2\}, \{3, 4\}, 5\}$

$Y = \{2, 3, 4, 5\}$

Here, $X \nsubseteq Y$ and $Y \nsubseteq X$ which imply $X \neq Y$.

Properties of Equality Sets

For any three sets X, Y and Z

(a) Reflexive $X = Y$

(b) Symmetric $X = Y \Rightarrow Y = X$

(c) Transitive $X = Y,\ Y = Z \Rightarrow X = Z$.

(iv) Proper Subset

For any set X if Y is the subset of X, but not equal to X, then Y is called proper subset of X. It is denoted by $Y \subset X$.

Symbolically $(Y \subseteq X \cap Y \neq X)$

Example

$X = \{1, 2, 3, 5, 7\}$

$Y = \{3, 5, 7\}$

$Y \subseteq X$

$Y \neq X \Rightarrow Y \subset X$.

(v) Universal Set

Any set which includes all the other sets defined is called universal set. It is denoted by μ or E.

Example

$A = \{x/x$ is a natural number$\}$

$B = \{x/x$ is a consonant$\}$

$C = \{x/x$ is an even prime number$\}$

$\mu = \{A, B, C\}$.

(vi) Null Set

A set with no element is called an empty or null set. It is denoted by ϕ or $\{\ \}$.

Note

For any set 'X', the null set 'ϕ' and the set X itself are subsets of set X.

(vii) Singleton Set

If a set consists of only one element then that set is referred as singleton set.

(viii) Power Set

If X is any set, then its power set consist of family of all its possible subsets. Power sets is denoted by $\rho^{(X)} = 2^X = \{x/x \subseteq x\}$.

Example

1. Let $X = \{1\}$

$\rho^{(X)} = \{\phi, \{1\}\} = \{\phi, X\}$

$\Rightarrow \ 2^1 = 2$

2. Let $Y = \{1, 2\}$

$\rho^{(X)} = \{\phi, \{1\}, \{2\}, \{1, 2\}\}$

$= \{\phi, \{1\}, \{2\}, S_2\}$

$\Rightarrow \ 2^2 = 4$

Note

If a set X contain n elements, then its power set contain 2^n elements.

(ix) Disjoint Set

If X and Y are two sets, which doesn't have any common elements (i.e., if $X \cap Y = \phi$) then these sets are said to be disjoint sets.

A collection of sets is said to be disjoint collection if there exist pair of disjoint sets. The elements belonging to disjoint collection are said to be mutually disjoint.

Q16. Explain with examples different operations that can be applied on the sets.

Answer :

The different operations performed on binary sets are as follows,

1. Intersection of Sets

If X and Y are two sets, then intersection of sets creates a new set that consists of the elements that are in X as well as in Y (i.e., common elements). Intersection of sets X and Y is denoted by $X \cap Y$.

Symbolically

$$A \cap B = \{x/(x \in A) \text{ and } (x \in B)\}$$

Properties of Intersection of Sets

(i) Commutative $X \cap Y = Y \cap X$

(ii) Associative $X \cap (Y \cap Z) = (X \cap Y) \cap Z$

(iii) $X \cap X = X$ (Idempotent)

(iv) $X \cap \phi = \phi$

2. Union of Sets

If X and Y are two sets, then union of set creates a new set that consist all elements present either in X or in Y or in both. Union of sets is denoted by $X \cup Y$.

Symbolically

$$X \cup Y = \{x/(x \in X) \text{ or } (X \in Y)\}$$

Properties of Union of Sets

(i) Commutative: $X \cup Y = Y \cup X$

(ii) Associative: $X \cup (Y \cup Z) = (X \cup Y) \cup Z$

(iii) $X \cup X = X$ (Idempotent)

(iv) $X \cup \phi = X$.

Example

$X = \{1, 2, 5, 7, 9\}$

$Y = \{3, 4, 5, 6, 8, 9, 7\}$

$X \cap Y = \{5, 7, 9\}$

$X \cup Y = \{1, 2, 3, 4, 5, 6, 7, 8, 9\}$

3. Relative Complement (Difference)

If X and Y are two sets, then relative complement of Y with respect to X, creates a new set that consist of elements belonging to X, but not Y. Relative complement is denoted by $X - Y$.

Symbolically

$$X - Y = \{x/(x \in X) \text{ and } (x \notin Y)\}$$

Properties of Relative Complement

(i) Commutative: $X - Y \neq Y - X$

(ii) Associative: $X - (Y - Z) \neq (X - Y) - Z$

Example

$X = \{1, 2, 5, 7, 9\}$

$Y = \{3, 4, 5, 6, 8, 9, 7\}$

$X - Y = \{1, 2, 5, 7, 9\} - \{3, 4, 5, 6, 8, 9, 7\}$

$= \{1, 2\}$

$Y - X = \{3, 4, 6, 8\}$

4. Symmetric Difference (Boolean Sum)

If X and Y are two sets, then their symmetric difference, creates a new set that consist of elements present either in A or in B but not both.

It is denoted as $X + Y$ or $X \Delta Y = (X - Y) \cup (Y - X)$.

Symbolically

$$X \Delta Y = \{x/(x \in A) \text{ or } (x \in B) \text{ but not both}\}$$

Properties of Symmetric Difference

(i) Commutative: $X + Y = Y + X$

(ii) Associative: $X + (Y + Z) = (X + Y) + Z$

(iii) Idempotent: $X + X = \phi$

(iv) $X + \phi = X$.

Example

$X = \{1, 2, 5, 7, 9\}$

$Y = \{3, 4, 5, 6, 8, 9, 7\}$

$X \Delta Y = (X - Y) \cup (Y - X)$

$= \{1, 2\} \cup \{3, 4, 6, 8\}$

$= \{1, 2, 3, 4, 6, 8\}$

Absolute Complement

$\mu - A$ (i.e., relative complement of A with respect to μ) is called as absolute complement or complement of A.

PROBLEMS ON BASICS OF SET THEORY

Q17. Show that for any two sets A and B

(i) $\rho(A) \cup \rho(B) \subseteq \rho(A \cup B)$

(ii) $\rho(A) \cap \rho(B) \subseteq \rho(A \cap B)$

Where $\rho(X)$ is the power set of X?

Solution :

Given that,

The $\rho(A)$ and $\rho(B)$ are power sets of sets A and B respectively.

To prove,

(i) $\rho(A) \cup \rho(B) \subseteq \rho(A \cup B)$

(ii) $\rho(A) \cap \rho(B) \subseteq \rho(A \cap B)$.

Proof

By definition, power set of any set is the set that contains all subsets of that set. Power set is denoted by 'ρ'.

$\rho(A)$ is set of all subsets of the set A, symbolically, we can write as,

$$\{a / a \subseteq A\} \text{(or) } 2^A$$

So, for $\rho(B)$, it can also be written as,

$$\{b / b \subseteq B\} \text{(or) } 2^B$$

(i) Let us prove this first statement with the help of an example, consider any two sets $A = \{1, 2, 3\}$, $B = \{1, 3, 6\}$

$\therefore$ $\rho(A) = \{0\}, \{1\}, \{2\}, \{3\}, \{1, 2\}, \{1, 3\}, \{2, 3\}, \{1, 2, 3\}$

$\therefore$ $\rho(B) = \{0\}, \{1\}, \{3\}, \{6\}, \{1, 3\}, \{1, 6\}, \{3, 6\}, \{1, 3, 6\}$

L.H.S $\Rightarrow \rho(A) \cup \rho(B)$

$= \{0\}, \{1\}, \{2\}, \{3\}, \{6\}, \{1, 2\}, \{1, 3\}, \{2, 3\}, \{1, 2, 3\}, \{1, 3, 6\}, \{1, 6\}, \{3, 6\}$ ... (1)

Now, find $A \cup B$ which is equal to $\{1, 2, 3, 6\}$

R.H.S $\Rightarrow \rho(A \cup B)$

$= \{0\}, \{1\}, \{2\}, \{3\}, \{6\}, \{1,2,3\}, \{1,2,6\}, \{2,3\}, \{2,6\}, \{2,3,6\}, \{3,6\}, \{1,2,3,6\}, \{1,3,6\}, \{1,6\}, \{1,2,6\}, \{1,3\}$ ... (2)

$= 2^4 = 16$ Subsets

$\therefore$ From equations (1) and (2), we get,

$\rho(A) \cup \rho(B) \subseteq \rho(A \cup B)$

(ii) To prove the second statement let us take the above example.

$\rho(A) \cup \rho(B) = \{0\}, \{1\}, \{3\}, \{1, 3\}$ ... (3)

Now, $A \cap B = \{1, 3\}$

$\therefore$ $\rho(A \cap B) = \{0\}, \{1\}, \{3\}, \{1, 3\}$ ... (4)

From equations (3) and (4), we get,

$\rho(A) \cap \rho(B) \subseteq \rho(A \cap B)$

Q18. Show that for any two sets A and B,

(a) $A - (A \cap B) = A - B$

(b) $A = A \cap B \cup (A - B)$

Solution :

(a) Given that,

Two sets are 'A' and 'B'

Require to prove, $A - (A \cap B) = A - B$

For the value y,

$y \in A - (A \cap B)$

$\Rightarrow$ $y \in \{y | y \in A \cap y \notin (A \cap B)\}$

$\Rightarrow$ $y \in A \cap \sim (y \in A \cap y \in B)$

$\Rightarrow$ $y \in A \cap (y \notin A \cup y \notin B)$

$\Rightarrow$ $(y \in A \cap y \notin A) \cup (y \in A \cap y \notin B)$

$\Rightarrow$ $(y \in A \cap y \notin A) \cup (y \in A \cap y \notin B)$

$\Rightarrow$ $y \in A \cap y \notin B$

$\therefore$ $y \in \{y | y \in A \cap y \notin B\}$

$\therefore$ $y \in A - B$

Hence proved.

(b) Given, two sets A and B,

Require to prove: $A = (A \cap B) \cup (A - B)$

Consider,

$$\text{R.H.S} = (A \cap B) \cup (A - B)$$

$$= x \in (A \cap B) \cup (A - B)$$

$$= x \in (A \cap B) \cup x \in (A \cap B')$$

$$= (x \in A \cap x \in B) \cup (x \in A \cap x \in B')$$

$$= x \in A \cap (x \in (B \cup B)$$

$$= x \in A \cap x \in U \qquad (\because A \cup A' = U)$$

$$= x \in (A \cap U) \qquad (\because A \cap U = A)$$

$$= A(\text{L.H.S})$$

Hence proved.

Q19. Prove that (A – B) ∪ (B – A) = (A∪B) – (A∩B) for any two sets A and B.

Solution :

Given, A and B and two sets,

To prove,

$$(A - B) \cup (B - A) = (A \cup B) - (A \cap B)$$

Let us initially prove,

(i) $(A - B) \cup (B - A) \subset (A \cup B) - (A \cap B)$

(ii) $(A \cup B) - (A \cap B) \subset (A - B) \cup (B - A)$

(i) $(A - B) \cup (B - A) \subset (A \cup B) - (A \cap B)$

Let $x \in (A - B) \cup (B - A)$

$\Rightarrow x \in (A - B)$ or $x \in (B - A)$

$\Rightarrow (x \in A$ or $x \notin B)$ or $(x \in B$ or $x \notin A)$

$\Rightarrow (x \in A$ or $x \in B)$ and $(x \notin B$ or $x \notin A)$

$\Rightarrow x \in (A \cup B) - (A \cap B)$

$\therefore (A - B) \cup (B - A) \subset (A \cup B) - (A \cap B)$ ... (1)

(ii) $(A \cup B) - (A \cap B) \subset (A - B) \cup (B - A)$

Let $x \in (A \cup B) - (A \cap B)$

$\Rightarrow x \in (A \cup B)$ and $x \notin (A \cap B)$

$\Rightarrow (x \in A$ or $x \in B)$ and $(x \notin A$ or $x \notin B)$

$\Rightarrow (x \in A$ or $x \notin B)$ and $(x \in B$ or $x \notin A)$

$\Rightarrow x \in (A - B) \cup (B - A)$

$\therefore (A \cup B) - (A \cap B) \subset (A - B) \cup (B - A)$ ... (2)

Thus, from equations (1) and (2), we get,

$$(A \cup B) - (A \cap B) = (A - B) \cup (B - A)$$

Hence proved.

4.3 PERMUTATION AND COMBINATION

Q20. Explain in detail about permutations.

Answer :

Permutations

Permutations refer to different arrangements of objects in a set where in all the elements are distinguishable. In these arrangements, an individual object is not repeated. If some objects are similar, then the permutations will be affected.

In other words, the different ways in which some objects (r) are selected and arranged from total number of objects (n) is called as "Permutations".

Example-1

If there are five different pens and three boxes, the number of arrangements of three pens in three boxes selected from five different pens is called as permutations.

The number of ways of arranging 'r' objects taken from 'n' different objects can be given by,

$$^nP_r = \frac{n!}{(n-r)!}$$

Where, '$n!$' is the number of ways of arranging 'n' objects.

$$n! = n(n-1)(n-2)(n-3)...$$

[$\because$ $n!$ is called as 'n' factorial, 0! is equal to 1]

For example,

$$5! = 5 \times 4 \times 3 \times 2 \times 1 = 120$$

$$6! = 6 \times 5 \times 4 \times 3 \times 2 \times 1 = 720$$

From the above example, for calculating the different ways of arranging of three pens from 5 different pens in three different boxes,

$$^nP_r = \frac{n!}{(n-r!)}$$

Here,

$n = 5$

$r = 3$

$$\therefore \quad ^5P_3 = \frac{5!}{(5-3)!} = \frac{5!}{2!}$$

$$= \frac{5 \times 4 \times 3 \times 2 \times 1}{2 \times 1} = 60$$

$\therefore$ In 60 ways three pens selected from five different pens can be arranged in three different boxes.

Example-2

A factory owner has received three distinguishable new machines A, B and C and these can be arranged in 6 ways in the following ways,

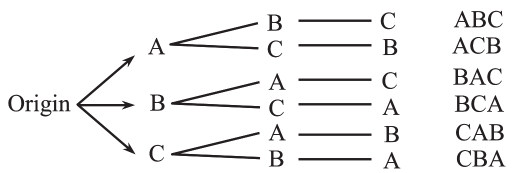
ABC, ACB, BAC, BCA, CAB, CBA

From the above, it should be noted that each arrangement has three elements and no element appears twice.

Example-3

The three distinguishable machines designated as A, B and C can be arranged on assembly line in the following manner.

First space | Second space | Third space | Passed arrangement



The better way to see how three spaces can be filled by the three different machines is,

Spaces	1^{st}	2^{nd}	3^{rd}
Ways	3	2	1

Applying the multiplication rule, the three spaces can be filled in,

$3 \times 2 \times 1 = 6$ ways, where repetition is not allowed.

Q21. Explain in detail about combinations.

Answer :

Combination

A combination refers to a selection of objects irrespective of their arrangement. The number of combinations of objects in different ways is entirely different from the number of their permutations.

The total number of possible combinations of a set of objects is always taken as 1.

For example, the possible arrangements from the set of objects 'a' and 'b' are ab and ba. Irrespective of their order ab is same as ba, there is only one possible combination for this set.

The number of ways of selecting 'r' objects from 'n' different objects irrespective of their arrangement is called as combinations.

It is denoted by ${}^nC_r$.

$$ {}^nC_r = \frac{n!}{r!(n-r)!} $$

Proof

Each combination of 'r' objects which can be arranged in $r!$ ways gives rise to $r!$ permutations.

Hence, permutations of each of the ${}^nC_r$ combination yield ${}^nC_r$ permutations.

We can thus, say that,

$$ {}^nC_r \times r! = {}^nP_r $$

$$ {}^nC_r = \frac{{}^nP_r}{r!} = \frac{\frac{n!}{(n-r)!}}{r!} $$

$$ \therefore \ {}^nC_r = \frac{n!}{r!(n-r)!} $$

Note

In permutations we are concerned with the arrangement of objects whereas in combinations we are not concerned with the arrangement of objects. Thus, only one combination is possible.

For example,

Combination	Permutations
ABC	ABC,ACB,BAC,BCA,CAB,CBA

In the above example of three distinguishable machines namely A, B and C, there is only one combination i.e., ABC, whereas in permutations we find six different types of arrangement as shown in the above table.

PROBLEMS ON PERMUTATION AND COMBINATION

Q22. How many ways are there to paste 2 photos on notice board from a group of 6 photos?

Solution : (Model Paper-III, Q12(a) | May/June-18, Q5(b) (KU))

Given,

Total Photos, $n = 6$

Photos to paste on the notice board, $r = 2$

$$ nP_r = \frac{r!}{(n-r)!} $$

$$ 6P_r = \frac{6!}{(6-2)!} = \frac{6!}{4!} $$

$$ = \frac{6\times5\times4\times3\times2\times1}{4\times3\times2\times1} $$

$$ = 6 \times 5 = 30 $$

$\therefore$ 30 ways are there to paste 2 photos on notice board from a group of 6 photos.

Q23. An urn contains 10 white, 4 red and 5 black balls. If 3 balls are drawn at random, find the probability that:

(i) Two of the balls drawn are white

(ii) One is of each colour

(iii) None is red

(iv) Atleast one is white.

Solution :

Given that,

urn contains 10 white(W), 4 red (R) and 5 black (B) balls

⇒ 10 + 4 + 5 = Total 19 Balls

⇒ Sample space, $S = {}^{19}C_3$

$${}^{19}C_3 = \frac{19!}{(19-3)!\,3!} = \frac{19!}{16! \times 3!} = \frac{19 \times 18 \times 17 \times 16!}{16! \times 3 \times 2 \times 1} = 969$$

(i) Probability that Two of the Balls Drawn are White

For two balls to be white, the third ball should be either red or black.

So, possible occurences should be WWR or WWB

$$\text{Now, probability} = \frac{\text{WWR} + \text{WWB}}{\text{S}}$$

$$= \frac{{}^{10}C_2\,{}^{4}C_1 + {}^{10}C_2\,{}^{5}C_1}{{}^{19}C_3}$$

$$= \frac{\left(\frac{10 \times 9 \times 8!}{8! \times 2}\right)\left(\frac{4 \times 3!}{3!}\right) + \left(\frac{10 \times 9 \times 8!}{8! \times 2}\right)\left(\frac{5 \times 4!}{4!}\right)}{969}$$

$$= \frac{(45)\,(4) + (45)\,(5)}{969}$$

$$= \frac{180 + 225}{969} = \frac{405}{969}$$

$= 0.418$ (or) 41.8%

(ii) Probability that One is of Each Colour

In this case, the balls drawn should be one white, one red and one black.

$$\text{So, Probability} = \frac{\text{W} \times \text{R} \times \text{B}}{\text{S}}$$

$$= \frac{{}^{10}C_1 \times {}^{4}C_1 \times {}^{5}C_1}{{}^{19}C_3}$$

$$= \frac{10 \times 4 \times 5}{969}$$

$$= \frac{200}{969}$$

$= 0.2064$ (or) 20.64%

(iii) Probability that None is Red

In this case, the three balls which are drawn should not contain a red ball (i.e.,) they should be either white or black.

10 white + 5 black balls = 15 balls

3 balls can be selected from white and black in ${}^{15}C_3$ ways.

$${}^{15}C_3 = \frac{15!}{12! \times 3!} = \frac{15 \times 14 \times 13 \times 12!}{12! \times 3 \times 2 \times 1} = 455$$

Now, probability that there is no red ball in three balls

$$= \frac{{}^{15}C_3}{{}^{19}C_3}$$

$$= \frac{455}{969}$$

$= 0.4696$ (or) 46.96%

(iv) Probability that Atleast One is White

Probability of drawing atleast one white ball = 1 – P (no white balls)

Probability of drawing no white balls can be calculated same as probability of drawing no red balls

$$\text{[case (iii)] P(No White Balls)} = \frac{{}^{9}C_3}{{}^{19}C_3}$$

$$= \frac{84}{969}$$

$= 0.0867$

∴ Probability of drawing atleast one white ball = 1 – 0.0867 = 0.9133 (or) 91.33%

Q24. The shoe company has sixty male employees and forty female employees. If two employees are selected at random, what is the probability that,

(i) Both will be males

(ii) Both will be females

(iii) There will be one of each gender.

The sum of these three probabilities will be one. Why?

Solution :

Number of males in the company = 60

Number of females in company = 40

Total number of employees = 100.

2 employees can be selected from 100 employees in ${}^{100}C_2$ ways.

(i) Both will be Males

Number of ways of selecting 2 male employees from 60 = ${}^{60}C_2$.

$$\text{Probability} = \frac{\text{Number of favourable cases}}{\text{Total number of exhaustive cases}}$$

∴ Probability of selecting two male employees

$$= \frac{{}^{60}C_2}{{}^{100}C_2} = \frac{60 \times 59}{100 \times 99} = \frac{3540}{9900} = \frac{59}{165} = 0.357$$

Therefore, if two employees are selected at random, the probability that both will be males 0.357 or 35.7%.

(ii) Both will be Females

Number of ways of selecting 2 females employees from 40 = ${}^{40}C_2$.

∴ Probability of selecting two females employees

$$= \frac{{}^{40}C_2}{{}^{100}C_2} = \frac{40\times39}{100\times99} = \frac{1560}{900} = \frac{26}{165} = 0.158 \text{ or } 15.8\%.$$

Therefore, if two employees are selected at random, the probability that both will be females = 0.158 or 15.8%

(iii) There will be One of Each Gender

1 male employee can be selected in ${}^{60}C_1$ ways 1 female employee can be selected in ${}^{40}C_1$ ways.

Since selecting a male and a female employees are independent events,

Probability that one male and one female will be selected $= \frac{{}^{60}C_1 \times {}^{40}C_1}{{}^{100}C_2} = \frac{60\times40\times2}{100\times99} = \frac{4800}{9900} = 0.485$ or 48.5%

Therefore, if two employees are selected at random, the probability that there will be one of each gender = 0.485 or 48.5%.

The sum of these three probabilities is equal to 1 i.e., 0.357 + 0.158 + 0.485 = 1.

The reason for sum of these probabilities equal to one is the three events are collectively exhaustive (i.e., it includes all the possible outcomes) and mutually exclusive (i.e., no two events can occur at the same time).

4.4 APPROACHES TO PROBABILTY: CLASSICAL, EMPIRICAL, SUBJECTIVE, AXIOMATIC

Q25. What are the various theories or approaches used in probability?

Answer :

Based on the concept of probability, there are four different approaches of probability. They are as follows,

1. Classical or Priori Approach

The classical or priori approach assumes that all the outcomes of a random experiment are mutually exclusive, and equally likely. According to priori approach in a random experiment when there are 'm' favourable cases when a favourable event A occurs and 'n' total possible cases when favourable event A does not occur. Then the probability of getting favourable cases $P(A)$ can be calculated as,

$$P(A) = \frac{\text{Number of favourable cases}}{\text{Total number of possible outcomes}} = \frac{m}{n}$$

Classical or priori approach is given by, "James Bernoulli". He was the first person who found the uncertainty quantitatively.

According to this approach,

(i) Probability of an event always lies between 0 and 1.

$$0 \le P(A) \le 1$$

(ii) Sum of probability of an event and its complement is always 1.

$$p + q = 1$$

⇒ Sum of the probability of success and failure is 1.

For example,

'p' is the probability of getting a head and

'q' is the probability of getting a tail.

Then,

$$p + q = \frac{1}{2} + \frac{1}{2} = 1$$

(iii) The probability of occurrence for any event has 3 chances.

(a) In case of certain event, $P(E) = 1$

(b) In case of impossible event, $P(E) = 0$

(c) In case of uncertain event, $0 \le P(E) \le 1$.

Limitations of Classical Theory

Classical theory does not holds good,

(a) When all the outcomes are not equally likely.

(b) When the collectively exhaustive events of an experiment are infinite.

(c) Classical theory does not provide answers to certain question which occurs in our daily life.

For example,

What is the probability of occurrence of rain now?

The chances of bulb getting failed etc.

2. Empirical or Relative Frequency Approach

Relative frequency approach was given by Richard Von Mises. In some cases, the desired event may or may not occur. This approach assumes that a random experiment is repeated number of times under identical conditions and the trials are independent to each other.

Relative frequency approach calculates the proportion of time known as relative frequency with which the event takes place repeatedly over an infinite number of times under identical conditions.

Therefore, the probability of occurring an event A, can be calculated as,

$$P(A) = \lim_{n \to \infty} \left\{ \frac{a}{n} \right\}$$

Where,

'a' is the number of times an event 'A' is repeated

'n' is the trials of an experiment.

As the probability of an event is ascertained by repetitive empirical observations, this probability is known as 'empirical probability'.

Limitations of Empirical or Relative Frequency Approach

Some of the limitations of this approach are,

(i) It takes large amount of time, as the experiments are repeated large number of times,

(ii) During the experimental time, the conditions may not be always identical and homogenous.

3. Subjective Approach

Subjective approach was initiated by "Frank Ramsey" in the year 1926 in his book, "The Foundation of Mathematics and Other Logical Essays".

Subjective approach depends on the extent to which the experiences focusses on the chances of the occurrence of random event. Hence, it is also known as 'personalistic approach'.

Limitations of Subjective Approach

Some of the limitations of this approach are,

(i) As the probability is just an estimation based on one's personal beliefs it may differ from one person to another person

(ii) We cannot calculate probability accurately as the results are not known

(iii) There is no particular formula for calculating probability quantitatively.

4. Axiomatic Approach

Axiomatic approach was given by the Russian mathematician, A.N.Kolmogorov in the year 1933 in his book "Foundation of Probability".

According to axiomatic approach, the probability of any event is calculated on basis of the axioms or postulates. Three axioms are taken into consideration for knowing the probability stated below,

(i) The probability of an event, if it does not occurs is 0(zero). For certain event it is 1(one) and for uncertain event it always ranges from 0 to 1

(ii) The probability of the whole sample space is always $P(S) = 1$

(iii) When 'A' and 'B' are two mutually exclusive events, then the probability of occurrence of either A or B is equal to the sum of their individual probabilities.

$$P(A \cup B) = P(A) + P(B)$$

PROBLEMS ON BASIC PROBABILITY

Q26. Two dice are rolled. Find the probability of 6 number event when two dice are rolled.

Solution : May/June-19, Q9(b) (MGU)

When two dice are rolled, the sample space, S contains $6^2 = 36$ outcomes as shown below,

$$S = \begin{Bmatrix} (1,1) & (1,2) & (1,3) & (1,4) & \mathbf{(1,5)} & (1,6) \\ (2,1) & (2,2) & (2,3) & \mathbf{(2,4)} & (2,5) & (2,6) \\ (3,1) & (3,2) & \mathbf{(3,3)} & (3,4) & (3,5) & (3,6) \\ (4,1) & \mathbf{(4,2)} & (4,3) & (4,4) & (4,5) & (4,6) \\ \mathbf{(5,1)} & (5,2) & (5,3) & (5,4) & (5,5) & (5,6) \\ (6,1) & (6,2) & (6,3) & (6,4) & (6,5) & (6,6) \end{Bmatrix}$$

Let 'E' be the event of getting 6.

Possible number of outcomes for the event 'E' is n(E) = 5

Probability of event 'E' $\Rightarrow$ p(E)= $\dfrac{n(E)}{n(S)}$

$$= \frac{5}{36}$$

$\therefore$ Probability of 6 number event when two dice are rolled is $\dfrac{5}{36}$

Q27. Calculate the probability of getting 52 sundays in a ordinary year and a leap year.

Solution :

(i) Probability of Getting 52 Sundays in a Ordinary Year

Case (i): Atleast 52 sundays

Ordinary year = 365 days

An ordinary year has 52 weeks

⇒ $52 \times 7 = 364$ days ⇒ There are 52 sundays for sure

∴ Probability of getting atleast 52 sundays is 1.

Case (ii): Exactly (or) only 52 sundays

1. There will be 1 day extra after 364 days/52 weeks.
2. That 1 day may be Sunday, Monday, Tuesday, Wednesday, Thursday, Friday or Saturday.
3. If the 365th day is Sunday, there will be 53 Sundays in a year. Probability of getting 53rd Sunday is 1 day out of 7 days. Thus, probability of getting 53rd Sunday is 1/7.

∴ Probability of getting exactly 52 Sundays = 1 – 1/7 = 6/7.

(i) Probability of Getting 52 Sundays in a Leap Year

Case (i): Atleast 52 Sundays

Leap Year = 366 days

A leap year also has 52 weeks

⇒ $52 \times 7 = 364$ days

⇒ There are 52 Sundays for sure

∴ Probability of getting atleast 52 Sundays is 1.

Case (ii): Exactly (or) only 52 Sundays

1. There will be 2 days extra after 364 days/52 weeks.
2. That 2 days may take the following combinations, **(Sunday, Monday)**, (Monday, Tuesday), (Tuesday, Wednesday), (Wednesday, Thursday), (Thursday, Friday), (Friday, Saturday), **(Saturday, Sunday)**.
3. If 365th days or 366th day is Sunday, there will be 53 Sundays. Probability of getting 53rd Sunday is 2 possible outcomes out of 7 possible outcomes. Thus, probability of getting 53rd Sunday is 2/7.

∴ Probability of getting exactly 52 Sundays = 1 – 2/7 = 5/7.

4.5 THEOREMS OF PROBABILITY: ADDITION, MULTIPLICATION

Q28. Explain the probability theorem basic concpets

May/June-19, Q9(a) (MGU)

OR

Explain the major theorems of probability.

Answer :

The major and important theorems of probability are as follows,

1. Addition Theorem

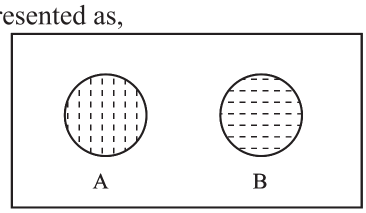
Addition theorem is different for mutually exclusive events and non-mutually exclusive events.

For Mutually Exclusive Events

When 'A' and 'B' are two mutually exclusive events (i.e., both cannot occur at the same time) then the probability of occurrence of A or B is equal to the sum of their individual probabilities.

$$P(A \text{ or } B) = P(A) + P(B)$$
$$\Rightarrow P(A \cup B) = P(A) + P(B)$$

Diagrametrically it can be represented as,



Mutually exclusive events

Figure: Mutually Exclusive Events

In case of 3 events A, B and C,

$$P(A \text{ or } B \text{ or } C) = P(A \cup B \cup C) = P(A) + P(B) + P(C)$$

For Non-Mutually Exclusive Events

In case of non-mutually exclusive event (i.e., if the events occur together) these is a variation in the addition theorem.

When 'A' and 'B' are non-mutually exclusive events then the probability of occurrence of A or B is the sum of their individual probability which should be deducted from the probability of A and B occurring together.

$$\boxed{P(A \text{ or } B) \Rightarrow P(A \cup B) = P(A) + P(B) - P(A \cap B)}$$

Diagrametically it can be represented as,

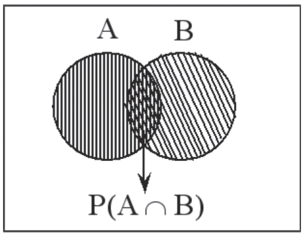


Figure: Non-Mutually Exclusive Events

In case of three non-mutually exclusive events.

A, B and C the probability of occurence of A or B or C can be calculated by the following formula,

$$\boxed{P(A \text{ or } B \text{ or } C) = P(A \cup B \cup C) = P(A) + P(B) + P(C) - P(A \cap B) - P(A \cap C) - P(B \cap C) + P(A \cap B \cap C)}$$

2. Multiplication Theorem

If 'A' and 'B' are two independent events then the probability of occurrence of both the events is equal to the product of their individual probabilities.

For independent events,

$$\boxed{P(A \cap B) = P(A).P(B)}$$

Similarly,

$$\boxed{P(A \cap B \cap C) = P(A).P(B).P(C)} \text{ and so on.}$$

If 'A' and 'B' are two dependent events, in such a case multiplication theorem is altered and is given as follows. For dependent events,

$$\boxed{\begin{aligned} P(A \cap B) &= P(A/B).P(B) \\ &= P(B/A).P(A) \end{aligned}}$$

Where, $P(A/B)$ is a conditional probability of A given that B has occurred (The probability of occurrence of event A when event B has already occurred is the conditional probability of A given B).

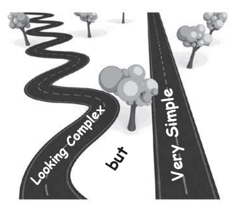
PROBLEMS ON THEOREMS OF PROBABILITY

Q29. A bag contains 4 defective and 6 good Electronic Calculators. Two Calculators are drawn at random one after the other without replacement. Find the probability that

(i) Two are good

(ii) Two are defective and

(iii) One is good and one is defective.



Solution : (Model Paper-II, Q12(b) | May/June-19, Q12(a) (OU))

Given that,

A Bag contains 4 defective and 6 good electronic Calculators,

$\Rightarrow$ Total number of calculators = 10

(i) Probability That Two are Good

Probability of getting good calculator in first draw is $\frac{6}{10} = \frac{3}{5}$

After one calculator is drawn, 9 calculators are left in the bag as the first one is not replaced. Now the probability of getting good calculator in second draw is $\frac{5}{9}$

$\therefore$ The probability of getting two good calculator is $\left(\frac{3}{5} \times \frac{5}{9}\right) = \frac{1}{3}$

(ii) Probability That Two are Defective

Probability of getting defective calculator in first draw is $\frac{4}{10} = \frac{2}{5}$

After one calculator is drawn, only 9 calculator are left in the bag. Now, the probability of getting defective calculator in second draw is $\frac{3}{9} = \frac{1}{3}$

$\therefore$ The probability of getting two defective calculator is $\frac{2}{5} \times \frac{1}{3} = \frac{2}{15}$

(iii) Probability That One is Good and One is Defective

Case (i)

First one is good and second one is defective

$$\text{Probability} = \frac{6}{10} \times \frac{4}{9} = \frac{4}{15}$$

Case (ii)

First one is defective and second one is good

$$\text{Probability} = \frac{4}{10} \times \frac{6}{9} = \frac{4}{15}$$

Probability of union of these two alternative cases is their sum

$$= \frac{4}{15} + \frac{4}{15} = \frac{8}{15}.$$

Q30. From 30 tickets marked with first 30 numerals, 1 ticket i6s drawn at random. It is then replaced and a second draw is made. Find the probability that in the first draw it is multiple of 5 or 7 and in second draw it is a multiple of 3 or 7.

Solution : May/June-18, Q12(a) (OU)

Calculation of Probability for Multiple of 5 or 7

Probability for a number to be a multiple of 5 is,

$$P(5, 10, 15, 20, 25, 30) = \frac{6}{30}$$

Probability for a number to be a multiple of 7 is,

$$P(7, 14, 21, 28) = \frac{4}{30}$$

There are no common events, therefore, the events are mutually exclusive.

$\therefore$ The probability of having a multiple of 5 or 7 is,

$$\frac{6}{30} + \frac{4}{30} = \frac{10}{30} = \frac{1}{3}$$

Calculation of Probability for Multiple of 3 or 7

Probability for a number to be a multiple of 3 is,

$$P(3, 6, 9, 12, 15, 18, 21, 24, 27, 30) = \frac{10}{30}$$

Probability for a number to be a multiple of 7 is,

$$P(7, 14, 21, 28) = \frac{4}{30}$$

As 21 is a multiple of both 3 and 7, the drawing of the ticked marked as 21 will result in occurrence of both events.

∴ The probability of having a multiple of 3 or 7 is,

$$\frac{10}{30} + \frac{4}{30} - \frac{1}{30} = \frac{13}{30}$$

Q31. The probability that a contractor will get a plumbing contract is 3/4 and the probability that he will not get electric contract is 4/9. If the probability of getting at least one contract is 5/6, what is the probability that he will get both the contracts? Use addition theorem to solve this problem.

Solution :

If I Don't Come, My Method Will Come

Let 'A' be the event that the contractor will get plumbing contract.

'B' be the event that the contractor will get electric contract.

The probability of getting plumbing contract = $P(A)$

The probability of not getting a plumbing contract = $P(\overline{A})$

The probability of getting electric contract = $P(B)$

The probability of not getting electric contract = $P(\overline{B})$

The probability of getting at least one contract is,

$$P(A \text{ or } B) = \frac{5}{6}$$

Given that,

$$P(A) = \frac{3}{4}; \; P(\overline{A}) = \frac{1}{4} \quad (\because P(A) + P(\overline{A}) = 1)$$

$$P(\overline{B}) = \frac{4}{9}; \; P(B) = \frac{5}{9}$$

∴ The probability that he will get both the contracts can be given by using addition theorem.

$$P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$$

$$P(A \text{ and } B) = P(A) + P(B) - P(A \text{ or } B)$$

$$= \frac{3}{4} + \frac{5}{9} - \frac{5}{6}$$

$$= \frac{27+20-30}{36} = \frac{47-30}{36} = \frac{17}{36} = 0.472 = 47.2\%$$

∴ The probability that the contractor will get both the contracts is = 47.2%

Q32. A husband and a wife appear in an interview for two vacancies in the same post. The probability of husband's selection is 1/9 and that of wife's selection is 1/5. What is the probability that,

(i) Both of them will be selected

(ii) Only one of them will be selected

(iii) None of them will be selected.

Use multiplication theorem to solve the problem.

Solution :

Let 'A' be the event of husband getting selected.

'B' be the event of wife getting selected.

Probability of husband's selection, $P(A) = \frac{1}{9}$

Probability of wife's selection, $P(B) = \frac{1}{5}$

(i) Both of Them Will be Selected

Since, events A and B are independent, probability of both of them getting selected,

$$P(A \cap B) = P(A).P(B)$$

[By multiplication theorem for independent events]

$$= \frac{1}{9} \times \frac{1}{5} = \frac{1}{45}$$

(ii) Only One of Them Will be Selected

Probability of only one of them will be selected is equal to the probability of husband being selected and wife not selected plus (+) probability of husband not selected and wife selected.

i.e., $P(A \cap B) = P(A).(P(\overline{B}) + P(\overline{A}).P(B)$

$$\because P(A) + P(\overline{A}) = 1$$

$$P(A) = \frac{1}{9}$$

$$P(\overline{A}) = 1 - \frac{1}{9} = \frac{8}{9}$$

$$P(B) = \frac{1}{5}$$

$$P(\overline{B}) = 1 - \frac{1}{5} = \frac{4}{5}$$

$$\therefore \quad P(A \cap B) = \frac{1}{9} \times \frac{4}{5} + \frac{8}{9} \times \frac{1}{5}$$

$$= \frac{4}{45} + \frac{8}{45} = \frac{12}{45} = \frac{4}{15}$$

$\therefore$ The probability of only one of them (either husband or wife) will be selected is $\frac{4}{15}$.

(iii) None of Them Will be Selected

The probability of none of them will be selected is calculated as follows,

$$P(\overline{A} \cap \overline{B}) = P(\overline{A}).P(\overline{B})$$

$$P(\overline{A}) = \frac{8}{9} \quad \text{... (1)}$$

$$P(\overline{B}) = \frac{4}{5} \quad \text{... (2)}$$

$$\therefore \quad P(\overline{A} \cap \overline{B}) = P(\overline{A}).P(\overline{B}) \quad \text{... (3)}$$

By substituting values of equations (1) and (2) in equation (3),

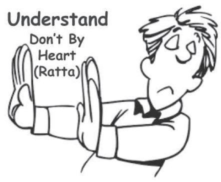
$$P(\overline{A} \cap \overline{B}) = \frac{8}{9} \times \frac{4}{5} = \frac{32}{45}$$

Therefore, the probability of none of them (neither husband nor wife) getting selected is $\frac{32}{45}$.

4.6 BAYE'S THEOREM

Q33. State and explain Baye's Probability theorem with its applications.

Answer :



Baye's Theorem

Baye's theorem got its name from the British mathematician, 'Thomas Bayes' in 1763. Baye's theorem deals with revising of priori probability by making use of new information for calculating posterior probabilities. It is also known as 'Rule for the Inverse Probability'.

In other words, priori probabilities are revised or converted into posterior probabilities (or revised probabilities) by using new information in Baye's theorem.

The probabilities of an event before the collection of new information is known as 'Priori probabilities'. Where as, Posterior probabilities are the revised priori probabilities that have been derived after using the new information. It is also known as 'Inverse' or 'Revised' probabilities.

If, $A_1, A_2, A_3 ... A_n$ are mutually exclusive and collectively exhaustive events, then

$P(A_1), P(A_2), P(A_3),..., P(A_n)$ are the priori probability,

'B' is an event such that $P(B) \neq 0$ whose conditional probabilities are represented as,

$$P(B/A_1), P(B/A_2), P(B/A_3),....., P(B/A_n)$$

It should be noted that the conditional probabilities are known with the help of this data we have to calculate posterior probabilities.

∴ Posterior probabilities can be calculated by using the following formula,

$$P(A_i / B) = \frac{P(A_i \cap B)}{\sum_{i=1}^{n} (A_i \cap B)} = \frac{P(B / A_i) P(A_i)}{\sum_{i=1}^{n} P(B / A_i)\ P(A_i)}$$

(or)

$$P(A_i / B) = \frac{P(A_i \cap B)}{P(B)}$$

Where,

$P(A_i \cap B)$ is the joint probability of A_i and B events

A_i and B events

$P(A_i)$ is the priori probability and

$P(B/A_i)$ is the conditional probability.

$$\therefore\ P(B) = \sum_{i=1}^{n} P(B / A_i)\ P(A_i)$$

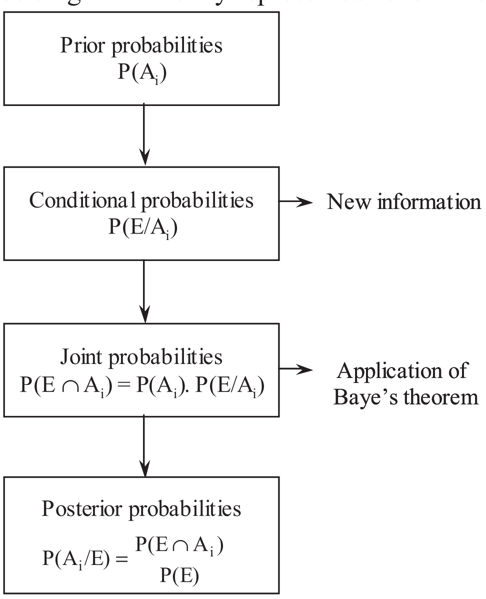
Applications of Baye's Theorem

The following points highlights the application of Baye's theorem,

1. In Baye's theorem, posterior probabilities can be known by revising priori probabilities with the help of new information
2. The probability of occurrence of future events can be known by Baye's theorem.
3. It offers a powerful statistical tool.
4. It helps the business and management executives to take effective decisions in uncertain situation.

Baye's theorem is also known as 'Probability of Causes' as it helps in determining the probability which a particular effect has due to a specific cause.

The entire revision process of priori probabilities can be diagrammatically represented as follows,



PROBLEMS ON BAYE'S THEOREM

Q34. A company has two Plants for manufacturing Scooters. Plant I manufactures 80% of the Scooters and Plant II manufactures 20%. At the Plant I 85% Scooters are rated to be of standard quality and at Plant II 65% Scooters are rated to be of standard quality. One Scooter was selected at random. What is the probability that

(i) It is manufactured by Plant I

(ii) It is manufactured by Plant II – which is of standard quality.

Solution : (Model Paper-I, Q12(b) | May/June-19, Q12(b) (OU))

Let,

The scooter produced by plant-I = A_1

The scooter produced by plant-II = A_2

The standard quality scooter produced by plant I and II = E

$$P(A_1) = 80\% \text{ or } 0.80$$

$$P(A_2) = 20\% \text{ or } 0.20$$

$$P(E/A_1) = \frac{85}{100} = 0.85$$

$$P(E/A_2) = \frac{65}{100} = 0.65$$

$$P(E \cap A_1) = P\left(\frac{E}{A_1}\right) \times P(A_1)$$

$$= 0.85 \times 0.80 = 0.68$$

$$P(E \cap A_2) = P\left(\frac{E}{A_2}\right) \times P(A_2)$$

$$= 0.65 \times 0.20 = 0.13$$

As per Baye's theorem the probability to manufacture standard quality scooter chosen at random

$$= P\left(\frac{A_1}{E}\right)$$

$$= \frac{P\left(\frac{E}{A_1}\right) \times P(A_1)}{P\left(\frac{E}{A_1}\right) \times P(A_1) + P\left(\frac{E}{A_2}\right) \times P(A_2)}$$

$$= \frac{0.68}{0.68 + 0.13}$$

$$= \frac{0.68}{0.81}$$

$$= 0.839 \text{ (or) } 0.84$$

$$= 0.84\% \text{ or } \frac{68\%}{81\%}$$

$$P\left(\frac{A_2}{E}\right) = \frac{P\left(\frac{E}{A_2}\right) \times P(A_2)}{P\left(\frac{E}{A_2}\right) \times P(A_2) + P\left(\frac{E}{A_1}\right) \times P(A_1)}$$

$$= \frac{0.13}{0.13 + 0.68} = \frac{0.13}{0.81}$$

$$= 0.16$$

$$= 0.16\% \text{ or } \frac{13\%}{81\%}$$

Q35. In a bolt factory, the Machines P, Q and R manufacture respectively 25%, 35% and 40% of the total of their outputs 5,4,2 percents respectively are defective bolts. A bolt is drawn at random from the product, and is known to be defective. What are the probabilities that it was manufactured by the machines P, Q and R.

Solution : **May/June-18, Q12(b) (OU)**

Let,

P(A) = The probability of the event that the bolt is manufactured by the machine P

P(B) = The probability of the event that the bolt is manufactured by the machine Q

P(C) = The probability of the event that the bolt is manufactured by the machine R

Given that,


I am complex but Worthy

$$P(A) = 25\,\% = \frac{25}{100} = \frac{1}{4}$$

$$P(B) = 35\,\% = \frac{35}{100} = \frac{7}{20}$$

$$P(C) = 40\,\% = \frac{40}{100} = \frac{2}{5}$$

Consider,

D = The event that the bolt drawn is defective.

P(D) = The probability of the event that the bolt drawn is defective.

Then,

$$P\left(\frac{D}{A}\right) = 5\% = \frac{5}{100}$$

$$P\left(\frac{D}{B}\right) = 4\% = \frac{4}{100}$$

$$P\left(\frac{D}{C}\right) = 2\% = \frac{2}{100}$$

The Probability that the bolt is defective manufactured from machine '*P*' is,

$$P\left(\frac{A}{D}\right) = \frac{P\left(\frac{D}{A}\right)P(A)}{P\left(\frac{D}{A}\right).P(A) + P\left(\frac{D}{B}\right).P(B) + P\left(\frac{D}{C}\right).P(C)}$$

$$P\left(\frac{A}{D}\right) = \frac{\frac{5}{100} \times \frac{1}{4}}{\left(\frac{5}{100} \times \frac{1}{4}\right) + \left(\frac{4}{100} \times \frac{7}{20}\right) + \left(\frac{2}{100} \times \frac{2}{5}\right)}$$

$$P\left(\frac{A}{D}\right) = \frac{\frac{1}{80}}{\left(\frac{1}{80}\right)+\left(\frac{7}{500}\right)+\left(\frac{1}{125}\right)}$$

$$P\left(\frac{A}{D}\right) = \frac{\frac{1}{80}}{\frac{69}{2000}}$$

$$\boxed{\therefore P\left(\frac{A}{D}\right) = \frac{25}{69}}$$

The Probability that the bolt is defective manufactured from machine 'Q' is,

$$P\left(\frac{B}{D}\right) = \frac{P\left(\frac{D}{B}\right)P(B)}{P\left(\frac{D}{A}\right)P(A)+P\left(\frac{D}{B}\right)P(B)+P\left(\frac{D}{C}\right)P(C)}$$

$$P\left(\frac{B}{D}\right) = \frac{\frac{4}{100}\times\frac{7}{20}}{\left(\frac{5}{100}\times\frac{1}{4}\right)+\left(\frac{4}{100}\times\frac{7}{20}\right)+\left(\frac{2}{100}\times\frac{2}{5}\right)}$$

$$P\left(\frac{B}{D}\right) = \frac{\frac{7}{500}}{\left(\frac{1}{80}\right)+\left(\frac{7}{500}\right)+\left(\frac{1}{125}\right)}$$

$$P\left(\frac{B}{D}\right) = \frac{\frac{7}{500}}{\frac{69}{2000}}$$

$$\boxed{\therefore P\left(\frac{B}{D}\right) = \frac{28}{69}}$$

The Probability that the bolt is defective manufactured from machine 'R' is,

$$P\left(\frac{C}{D}\right) = \frac{P\left(\frac{D}{C}\right)P(C)}{P\left(\frac{D}{A}\right)P(A)+P\left(\frac{D}{B}\right)P(B)+P\left(\frac{D}{C}\right)P(C)}$$

$$P\left(\frac{C}{D}\right) = \frac{\frac{2}{100}\times\frac{2}{5}}{\left(\frac{5}{100}\times\frac{1}{4}\right)+\left(\frac{4}{100}\times\frac{7}{20}\right)+\left(\frac{2}{100}\times\frac{2}{5}\right)}$$

$$P\left(\frac{C}{D}\right) = \frac{\frac{1}{125}}{\left(\frac{1}{80}\right)+\left(\frac{7}{500}\right)+\left(\frac{1}{125}\right)}$$

$$P\left(\frac{C}{D}\right) = \frac{\frac{1}{125}}{\frac{69}{2000}}$$

$$\boxed{\therefore P\left(\frac{C}{D}\right) = \frac{16}{69}}$$

Q36. In a certain college 25% of boys and 10% of girls are studying mathematics. The girls constitute 60% of the students. If a student is selected at random and is found to be studying mathematics, find the probability that the student is a (i) girl (ii) boy.

Solution : Model Paper-III, Q12(b)

Let,

$E_1 \rightarrow$ Boy is selected

$E_2 \rightarrow$ Girl is selected

$A \rightarrow$ Denote the event that the mathematics is studied.

Given that,

$P(E_1)$ = The probability that the boy is selected

$$= \frac{40}{100} = \frac{2}{5}$$

$P(E_2)$ = The probability that the girl is selected

$$= \frac{60}{100} = \frac{3}{5}$$

$P(A/E_1)$ = Probability that mathematics is studied by boy

$$= \frac{25}{100} = \frac{1}{4}$$

$P(A/E_2)$ = Probability that mathematics is studied by girl

$$= \frac{10}{100} = \frac{1}{10}$$

Probability that mathematics is studied is given from the theorem of total probability,

$$P(A) = \sum_{i=1}^{2} P(E_i)P(A/E_i)$$

$$= P(E_1).P(A/E_1) + P(E_2).P(A/E_2)$$

$$= \frac{2}{5}.\frac{1}{4} + \frac{3}{5}.\frac{1}{10} = \frac{4}{25}$$

(i) Probability that a mathematics student is a girl $= P(E_2/A)$ [from Baye's theorem]

$$\therefore P(E_2/A) = \frac{P(E_2).P(A/E_2)}{P(A)} = \frac{\frac{3}{5}.\frac{1}{10}}{\frac{4}{25}} = \frac{3}{8}$$

(ii) Probability that a mathematics student is a boy $= P(E_1/A)$

$$\therefore \quad P(E_1/A) = \frac{P(E_1).P(A/E_1)}{P(A)}$$

[From Baye's theorem]

$$= \frac{\frac{2}{5}.\frac{1}{4}}{\frac{4}{25}} = \frac{5}{8}$$

Q37. A manufacturing firm produces pipes in three plants, plant A produces 50% of total output, plant B produces 25% and plant C produces 25% of total output. From the past experience the fraction of defective items in the output of these plants were 0.005, 0.008, 0.010. Find out the probability of a defective item selected at random is from A, B and C plants.

Solution :

Let 'A' be the total volume of production produced by a plant 'A'.

'B' be the production volume produced by plant B and

'C' be the production volume produced by plant C

Probability of production of pipes by plant A,

$P(A) = 50\% = 0.5$ (given)

Probability of production of pipes by plant B,

$P(B) = 25\% = 0.25$ (given)

Probability of production of pipes by plant C,

$P(C) = 25\% = 0.25$ (given)

Let 'D' be the number of defective items produced by all the plants.

$\therefore$ The probability of defective items produced by plant A,

$P(D/A) = 0.005$ (given)

Probability of defective items produced by plant B,

$P(D/B) = 0.008$

Probability of defective items produced by plant C,

$P(D/C) = 0.01$

Now calculating the joint probabilities,

Plant A, $P(A \cap D) = P(A) \times P(D/A)$

$= 0.5 \times 0.005 = 0.0025$

Plant B, $P(B \cap D) = P(B) \times P(D/B)$

$= 0.25 \times 0.008 = 0.002$

Plant C, $P(C \cap D) = P(C) \times P(D/C)$

$= 0.25 \times 0.01 = 0.0025$

For getting the required probabilities, Baye's theorem is applied in this case.

Computing the values by using Baye's theorem is tabulated as follows,

Event (1)	Probability (2)	Conditional Probability (3)	Joint Probability (4) = (2) × (3)	Posterior Probability (5) = $\frac{(4)}{0.007}$
A	$P(A) = 0.5$	$P(D/A) = 0.005$	$P(A \cap D) = 0.5 \times 0.005$ $= 0.0025$	$P(A/D) = \frac{0.0025}{0.007}$ $= 0.357$
B	$P(B) = 0.25$	$P(D/B) = 0.008$	$P(B \cap D) = 0.25 \times 0.008$ $= 0.002$	$P(B/D) = \frac{0.002}{0.007}$ $= 0.286$
C	$P(C) = 0.25$	$P(D/C) = 0.01$	$P(C \cap D) = 0.25 \times 0.01$ $= 0.0025$	$P(C/D) = \frac{0.0025}{0.007}$ $= 0.357$
Total	**1**		**0.007**	**1**

$\therefore$ The probability of a defective item selected at random is,

From plant $A = 0.357 = 35.7\%$

From plant $B = 0.286 = 28.6\%$

From plant $C = 0.357 = 35.7\%$

EXERCISE AND PRACTICE QUESTIONS

SHORT QUESTIONS

THEORY

Q1. Define Probability. [Refer, Q1]

Q2. What are mutually exclusive events, non-mutually exclusive events and dependent events?

OR

Explain (i) Mutually exclusive events and (ii) Not-mutually exclusive events.

May/June-18, Q5 (OU)

OR

Explain:

(i) Mutually exclusive events and

(ii) Dependent events. [Refer, Q2] May/June-19, Q6 (OU)

Q3. What do you mean by Conditional Probability? [Refer, Q3]

Q4. What is Joint Probability? [Refer, Q4]

Q5. What is Marginal Probability? [Refer, Q5]

PROBLEMS

Q6. $n(A) = 70$, $n(B) = 60$, $n(A \cap B) = 40$ then find $\cap (A \cup B)$. [Refer Similar, Q8]

(Ans: $n(A \cup B) = 90$).

Q7. How many 5 letter words can be formed from the English world "ACCESS"? [Refer Similar, Q9]

Q8. Find the value of $8P_6$, $6P_4$. [Refer Similar, Q10]

Q9. Calculate probability of 53 Tuesdays in a leap year. [Refer Similar, Q11]

ESSAY QUESTIONS

THEORY

Q10. What do you mean by probability? Explain the importance of probability. [Refer, Q12]

Q11. What are the key concepts of probability? [Refer, Q13] May/June-18, Q5(a) (KU)

Q12. Define set. How is it denoted and what are the two different ways of representing a set along with an example? [Refer, Q14]

Q13. Explain with examples different operations that can be applied on the sets. [Refer, Q16]

Q14. Explain in detail about permutations. [Refer, Q20]

Q15. What are the various theories or approaches used in probability? [Refer, Q25]

Q16. Explain the probability theorem basic concpets. [Refer, Q28] May/June-19, Q9(a) (MGU)

Q17. State and explain Baye's Probability theorem with its applications. [Refer, Q33]

PROBLEMS

Q18 In a certain class there are 21 students in subject X, 17 in subject Y and 10 in subject Z. Of these 12 attend subjects X and Y, 5 attend subjects Y and Z, 6 attend subject X and Z. These include 2 students who attend all the three subjects. Find the probability that a student studies one subject alone. [Refer Similar Q31]

(Ans: 8/27).

Q19. A box contains 4 red pens and 5 black pens. Find the probability of drawing 3 black pens one by one, [Refer Similar Q23]

(i) With replacement (ii) Without replacement.

(Ans: (i) 125/729, (ii) 5/42).

Q20. Consider a standard deck of cards. One card is drawn at random. Find the probability of drawing either a king or a red card. [Refer Similar Q7]

(Ans: 7/13).

Q21. A manufacturing firm produces steel pipes in three plants with daily production volumes of 500, 1000 and 2000 units respectively. According to past experience, it is known that the fraction of defective outputs produced by the three plants are respectively 0.005, 0.008, 0.010. If a pipe is selected from the day's total production and found to be defective, find out,

From which plant the pipe comes? (or)

What is the probability that it comes from the first plant, 2nd plant and third plant? [Refer Similar Q37]

(Ans: Total Production = 3,500 Units; Plant A = 8.27%, Plant B = 26.18%, Plant C = 65.55%).

Q22. In a factory, machine A produces 40% of the output and machine B produces 60%. On the average, 9 items in 1000 produced by A are defective and 1 item in 250 produced by B is defective. An item drawn at random from a day's output is defective. What is the probability that it was produced by A or B? [Refer Similar Q34]

(Ans: A = 0.6, B = 0.4).

Q23. If A and B are two sets, then define $A \triangle B = (A - B) \cup (B - A)$. If A = {1, 2, 3} B = {1, 3, 5} then find the set

$((A \triangle B) \triangle B) - (A \triangle (B \triangle B))$. [Refer Similar Q16]

(Ans: $(A \triangle B) \triangle B = \{1, 2, 3\}$, $A \triangle (B \triangle B) = \{1, 2, 3, \phi\}$).

Q24. If A = {1, 2, 3} B = {2, 3, 4} C = {1, 3, 4} then find, [Refer Similar Q16]

(i) $A - (B - C)$ (ii) $A - (B \cap C)$

(iii) $(A \cup B) - C$.

(Ans: $A - (B - C) = \{1, 3\}$, $A - (B \cap C) = \{1, 2\}$, $(A \cap B) - C = \{2\}$).

INTERNAL ASSESSMENT/EXAM

I Multiple Choice

1. The term 'probability' was first coined by an Italian Mathematician, _________. [ ]
   (a) Flintof
   (b) Steven Smith
   (c) Galileo
   (d) Gayle
2. _________ is also known as 'mathematical probability'. [ ]
   (a) Classical probability
   (b) Statistical probability
   (c) Axiomatic probability
   (d) Chain probability
3. The collection of finite or infinite number of objects with some common property is called as _________. [ ]
   (a) Set
   (b) Pair
   (c) Probability
   (d) Method
4. In _________ the elements are enclosed with in the "{ }" brackets. [ ]
   (a) Cutter method
   (b) Disaster method
   (c) Roaster method
   (d) Toaster method
5. _________ is different for mutually exclusive events and non-mutually exclusive events. [ ]
   (a) Addition theorem
   (b) Multiplicative theorem
   (c) Division theorem
   (d) Substractive theorem
6. Baye's theorem got its name from the British mathematician, _________. [ ]
   (a) Andrew Bayes
   (b) Ricky Bayes
   (c) Chris Bayes
   (d) Thomas Bayes

7. Priori probabilities are revised or converted into _________ probabilities. [ ]

   (a) Interior

   (b) Posterior

   (c) Exterior

   (d) In-exterior

8. _________ theorem offers a powerful statistical tool. [ ]

   (a) Ross

   (b) Jos

   (c) Baye's

   (d) Gross

9. Baye's theorem is also known as _________. [ ]

   (a) Probability of cures

   (b) Probability of causes

   (c) Probability of cases

   (d) Probability of posses

10. Two or more events are considered as _________ event. [ ]

   (a) Independent

   (b) Dependent

   (c) Republic

   (d) Transformational

II Fill in the Blanks

1. Statistical probability is also known as __________ probability.
2. Probability helps in taking effective decisions under __________ conditions.
3. The result of a random experiment is usually referred as an __________.
4. When the joint occurrence of two or more events is considered then it is known as __________.
5. Axiomatic approach was given by __________.
6. Subjective approach was initiated by __________.
7. The major theorems of probability are __________ and __________ theorems.
8. __________ probability is also known as 'single probability'.
9. The probability of certain event is __________.
10. A set is usually denoted by capital letters with or without __________.

KEY

I. Multiple Choice

1. (c)
2. (a)
3. (a)
4. (c)
5. (a)
6. (d)
7. (b)
8. (c)
9. (b)
10. (a)

II. Fill in the Blanks

1. Relative frequency probability
2. Uncertain
3. Out come
4. Compound event
5. A.N. Kolmogorov
6. Frank Ramsey
7. Addition, Multiplicative
8. Marginal probability
9. 1
10. Subscripts.

III Very Short Questions and Answers

Q1. Define probability.

Answer :

Probability can be defined as the chance or 'likelihood of occurrence' of an experiment or event. Probability of any event ranges from 0 to 1. The term 'probability' was coined by an Italian mathematician, 'Galileo'. He was the first man who measured probability quantitatively while dealing with the problems associated with rolling of dice in gambling.

Q2. What is Experiment?

Answer :

An experiment is also referred as random experiment. It is a process or activity which leads to a particular outcome of several possible outcomes. The outcome which is going to be derived through random experiment is not known until it's occurence (i.e., the outcome of random experiment is not predictable). But, the number of possible outcomes can be known. There may be fixed or infinite number of outcomes for a particular experiment. Outcome from random experiment may be numerical or non-numerical in nature.

Q3. What do you understand by set?

Answer :

The collection of finite or infinite number of objects with some common property is called Set. The objects belonging to the set are called Members or Elements of the set.

Q4. Write a note on Baye's theorem.

Answer :

Baye's theorem got its name from the British mathematician, 'Thomas Bayes' in 1763. Baye's theorem deals with revising of priori probability by making use of new information for calculating posterior probabilities. It is also known as 'Rule for the Inverse Probability'.

Q5. What are Complementary Events?

Answer :

Two events are said to be complementary events if they are mutually exclusive and collectively exhaustive.

For example, when a coin is tossed, getting a head or a tail are mutually exclusive and collectively exhaustive. Hence, if we get tail then head is considered as its complementary event.

UNIT 5

Theoretical Distributions

SYLLABUS

Binomial Distribution: Importance – Conditions – Constants - Fitting of Binomial Distribution. Poisson Distribution: – Importance – Conditions – Constants - Fitting of Poisson Distribution. Normal Distribution: – Importance - Central Limit Theorem - Characteristics – Fitting a Normal Distribution (Areas Method Only).

LEARNING OBJECTIVES

- ✓ *The Concept of Binomial Distribution with its Importance, Properties, Applications and Assumptions.*
- ✓ *Conditions and Constants under Binomial Distribution.*
- ✓ *Fitting of Binomial Distribution.*
- ✓ *The Concept of Poisson Distribution with its Importance, Conditions and Constants.*
- ✓ *Fitting a Poisson Distribution.*
- ✓ *The Concept of Normal Distribution with its Importance and Characteristics.*
- ✓ *The Concept of Central Limit Theorem.*
- ✓ *Fitting a Normal Distribution.*

INTRODUCTION

Binomial distribution is a discrete probability distribution developed by a Swiss mathematician, 'James Bernoulli' in 1700. Thus, it is also known as Bernoulli distribution. Binomial distribution is used for finite or limited number of trails 'n'. It produces successes and failures based on two parameters 'n' and 'p'.

Poisson distribution is a discrete probability distribution for countably infinite trials. Poisson distribution is named after French mathematician, 'SIMEON DENIS POISSON' in 1837. It is used when the probability of success of any individual event is very small. The average or mean of poisson distribution is given by λ. However, the single parameter of poisson distribution is also given as λ. Poisson distribution can be used generally to approximate the binomial distribution when 'n' value is large (i.e., $n \to \infty$) and 'p' value is very small (i.e., $p \to o$).

Normal distribution was first discovered by 'Abraham Demoivre' in 1733 as a limiting case of binomial distribution. It was later developed by LAPLACE and GAUSS. Normal distribution is also known as 'Gaussian distribution' as the credit goes to German mathematician 'karl friedrich gauss'. In normal distribution, the probability of occurrence of values of random variables are calculated within a range or interval whereas the probability of a particular value cannot be calculated and is always assumed to be zero. The probability distribution used mostly for dealing with quantities whose magnitude is continuously variable is normal distribution. Hence, it is called as 'Normal probability distribution'.

Central limit theorem states that the distribution of the sum of I.I.D (Independently and Identically Distributed) random variables will be normal asymptotically under general conditions with mean (μ) and standard deviation (σ).

PART-A SHORT QUESTIONS AND ANSWERS

Q1. Write the Mean of Binomial Distribution.

Answer :

The mean of binomial distribution is denoted by 'μ' or '$E(X)$' is the expected number of successes in 'n' number of trials.

$\therefore$ The mean of binomial distribution,

$$\boxed{\mu = np}$$

Where,

'n' = Number of trials

'p' = Probability of success in a single trial.

The two parameters of binomial distribution are 'n' and 'p'.

Q2. Bring out the differences between Binomial and Poisson distribution.

Answer :

Differences between Binomial and Poisson distribution are as follows,

S.No.	Basis	Binomial Distribution	Poisson Distribution
1.	Uses	It is used for finite or limited number of trails 'n'.	It is used where 'n' $\rightarrow \infty$ and probability of success $P \rightarrow 0$.
2.	Parameters	It has two parameters 'n' and 'p'.	It has a single parameter 'λ' (where $\lambda = np$).
3.	Mean and Variance	In this, mean $\mu = np$ and variance $\sigma^2 = npq$.	In this, mean and variance are equal to $\boxed{\lambda = \sigma^2}$ or $\boxed{\sigma = \sqrt{\lambda}}$.
4.	Success and Failures	It produces successes and failures.	It produces successes commonly referred as 'occurrences'.

Q3. Write about the relation between normal and binomial distribution.

Answer :

Binomial distribution can be closely approximated to normal distribution under certain conditions which are as follows,

1. When the number of trials 'n' is very large, $n \rightarrow \infty$.
2. Either 'p' or 'q' is too close to zero.

Thus, the standardized random variable is given by,

$$\boxed{z = \frac{x - np}{\sqrt{npq}}}$$

'z' will follow normal distribution with mean 'zero' and variance 'one'.

Q4. Comment on the following:

For a Binomial Distribution Mean = 7 and Variance = 11.

Answer : (Model Paper-I, Q8 | May/June-19, Q8 (OU))

For a Binomial Distribution,

Mean = np = 7 ......(1)

Variance = npq = 11 ......(2)

Dividing (2) by (1)

$$\frac{npq}{np} = \frac{11}{7}$$

$q = 1.57$ (or) 1.6

The value of q (probability) should not be more than 1. It should lie between 0 and 1. Therefore, it can be concluded that, the given data is inconsistent.

Q5. Properties of Normal Distribution.

Answer : (Model Paper-I, Q7 | May/June-18, Q8 (OU))

Following are the characteristics/features/properties of normal distribution,

1. The normal curve is 'bell-shaped' and symmetrical about the mean (skewness = 0). If the curve is folded along its central vertical axis the curves either side of the axis would coincide.
2. The height of the normal curve is maximum at its mean. Hence, the mean and mode coincide. Thus, in normal distribution, mean, mode and median are equal.
3. The height of the curve is maximum at its mean but reduces as it goes towards either of the direction but never touches the base. Hence, the curve is known as ASYMPTOTIC. The range in unlimited or infinite in both the directions.
4. As there is only one maximum point, the normal curve has only one mode and it known as 'unimodal'.
5. The points of inflexion i.e., the points where the change in curvature occurs are $\bar{x} \pm \sigma$ (or) $\mu \pm \sigma$.

Q6. 6 coins are tossed at the same time find the probability that 4 heads are occurred.

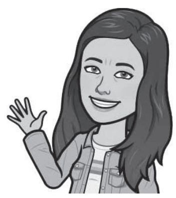
Answer : May/June-19, Q5 (MGU)

Given that,

Total number of coins tossed at a time, n = 6

Probability of getting head $P(H) = \frac{1}{2}$

Probability of getting tail $P(T) = \frac{1}{2}$



Keep an eye on me

Probability of getting 4 heads $P(H = 4) = {}^{n}C_{r}\left(\frac{1}{2}\right)^{r}\left(\frac{1}{2}\right)^{n-r}$

$$= {}^{6}C_{4}\left(\frac{1}{2}\right)^{4}\left(\frac{1}{2}\right)^{6-4}$$

$$= \frac{6!}{2!\times 4!}\times\left(\frac{1}{2}\right)^{4}\left(\frac{1}{2}\right)^{2}$$

$$= \frac{6\times 5\times 4!}{2!\times 4!}\times\left[\frac{1}{2}\right]^{4}\left[\frac{1}{2}\right]^{2}$$

$$= 15\times\left(\frac{1}{2}\right)^{6}$$

$$= 15\times\left[\frac{1}{64}\right]$$

$$= \frac{15}{64}$$

$\therefore$ The probability of getting 4 heads $P(H = 4) = \frac{15}{64}$

Q7. 6 coins are tossed at a time, what is the probability of obtaining 4 or more heads?

Answer : (Model Paper-II, Q8 | May/June-18, Q7 (OU))

Given that,

Total number of coins tossed at a time = 6

$\therefore n = 6$

Probability of getting a head, $p = \frac{1}{2}$

Probability of not getting a head $q = 1 - p$

$$= 1 - \frac{1}{2}$$

$$= \frac{2-1}{2}$$

$$\therefore q = \frac{1}{2}$$

We know that the binomial distribution is given by,

$$P(X = r) \ = {}^nC_r\, p^r\, q^{n-r}$$

Therefore, the probability of obtaining 4 or more heads is,

$$P(X \geq 4) = P(X = 4) + P(X = 5) + P(X = 6)$$

$$= 6_{c_4}\left(\frac{1}{2}\right)^4\left(\frac{1}{2}\right)^{6-4} + 6_{c_5}\left(\frac{1}{2}\right)^5\left(\frac{1}{2}\right)^{6-5} + 6_{c_6}\left(\frac{1}{2}\right)^6\left(\frac{1}{2}\right)^{6-6}$$

$$= 15\left(\frac{1}{2}\right)^4\left(\frac{1}{2}\right)^2 + 6\left(\frac{1}{2}\right)^5\left(\frac{1}{2}\right)^1 + 1\left(\frac{1}{2}\right)^6\left(\frac{1}{2}\right)^0$$

$$=15\left(\frac{1}{16}\right)\left(\frac{1}{4}\right) + 6\left(\frac{1}{32}\right)\left(\frac{1}{2}\right) + 1\left(\frac{1}{64}\right)$$

$$= \frac{15}{64} + \frac{6}{64} + \frac{1}{64} = \frac{15+6+1}{64} = \frac{22}{64}$$

$$\therefore P(X \geq 4) = \frac{11}{32} \text{ or } 0.34$$

Q8. What is the probability of getting 3 heads when a coin is tossed 5 times?

Answer : **(Model Paper-III, Q8 | May/June-18, Q1(h) (KU))**

Given that,

Number of tosses, $n = 5$

Probability of getting a head, $P = \frac{1}{2}$

Probability of not getting a head, $q = 1 - p = 1 - \frac{1}{2} = \frac{1}{2}$

Probability of getting 3 heads $= {}^nC_r \left(\frac{1}{2}\right)^r\left(\frac{1}{2}\right)^{n-r}$

$$= {}^5C_3 . \left[\frac{1}{2}\right]^3\left[\frac{1}{2}\right]^{5-3}$$

$$= \frac{5!}{3! \times 2!} \times \left[\frac{1}{2}\right]^3\left[\frac{1}{2}\right]^2$$

$$= \frac{5{\times}4{\times}3!}{3! \times 2 \times 1} \times \left[\frac{1}{2}\right]^5 = 10 \times \left[\frac{1}{2}\right]^5$$

$$= 10 \times \frac{1}{32} = \frac{5}{16}$$

$\therefore$ Probability of getting 3 heads when coin is tossed 5 times $= \frac{5}{16}$ or 0.3125

PART-B ESSAY QUESTIONS AND ANSWERS

5.1 BINOMIAL DISTRIBUTION – IMPORTANCE, PROPERTIES, APPLICATIONS AND ASSUMPTIONS

Q9. What is Binomial distribution? State its importance, applications and assumptions.

Answer : Model Paper-I, Q13(a)

Binomial Distribution

Binomial distribution is a discrete probability distribution developed by a Swiss Mathematician, 'James Bernoulli' in 1700. Thus, it is also known as Bernoulli distribution. It is used for finite or limited number of trials 'n'. It produces successes and failures based on two parameters 'n' and 'p'.

Binomial distribution satisfies two essential properties of probability distribution which are explained as follows,

(i) f(x) $\geq$ 0

Binomial distribution fulfills this requirement as 'n' and 'p' both are positive.

Therefore, ${}^nC_r\, p^r\, q^{n-r}$ are all positive.

So $\boxed{f(x) \geq 0}$

(ii) Σf(x) = 1

Binomial expansion of $(p+q)^n$ helps in fulfilling this requirement.

$$\text{As } \Sigma f(x) = \Sigma\, {}^nC_r . p^r q^{n-r} \qquad [\because p = 1 - q]$$

$$= (p+q)^r \Rightarrow (1 - q + q)^r$$

$$= (1)^r = 1 \Rightarrow \boxed{\Sigma f(x) = 1}$$

Importance of Binomial Distribution

The following points highlight the importance of binomial distribution,

1. It is an extensively used probability distribution of a discrete random variable.
2. It plays a vital role in the functions of quality control and quality assurance.
3. It is used in manufacturing units for defective analysis.
4. It is also used in service organizations like banks and insurance companies.
5. It characterizes the outcomes of each trials in the process on one of two types of possible outcomes.
6. The possibility of outcome of any trials does not change and remains independent compared to previous trials.

Applications of Binomial Distribution

Binomial distribution is applicable in case of repeated trials such as,

1. Number of applications received for a junior assistant post during a particular period of time.
2. Number of births taking place in a hospital.
3. Number of candidates appearing for the screening test conducted by a company.

All the trials are statistically independent and each trial has two outcomes namely, success and failure.

Assumptions of Binomial Distribution

Binomial distribution is assumed under the following conditions,

1. The number of trials 'n' is fixed and finite.
2. Trials are independent of each other.
3. Probability of success 'p' is constant for each trial and also the probability of failure 'q' is constant for each trial. Always $\boxed{p+q=1}$
4. Each trial has only two possible outcomes as success and failure.

Q10. Explain about the properties, mean and variance of binomial distribution.

Answer :

Properties of Binomial Distribution

The properties of Binomial distribution are as follows,

1. It describes the distribution of probabilities when there are only two mutually exclusive outcomes for each trial of an experiment for example while tossing a coin, the two possible outcomes are head and tail.
2. The process is performed under identical conditions for 'n' number of times.
3. Each trial is independent of other trials. It means the outcome of a particular trial does not affect the outcome of another trial.
4. The probability of success 'p' remains same for trial to trial throughout the experiment and similarly, the probability of failure ($q = 1 - p$) also remains constant overall the observations.
5. Binomial distribution is symmetrical when $p = 0.5$ [figure (i)] and it is skewed if $p \neq 0.5$, where 'n' can be any value.

   When $p > 0.5$ [figure (iii)], it is skewed to the right $\rightarrow$ negatively skewed.

   When $p < 0.5$ [figure (ii)], it is skewed to the left $\rightarrow$ positively skewed.

   Hence, binomial distribution is 'Asymmetrical'

   When $p > 0.5$ and $p < 0.5$

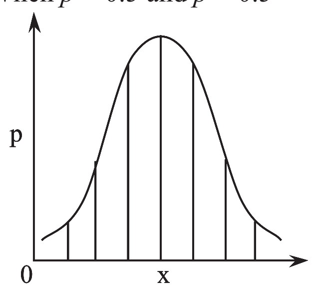


Figure (i)

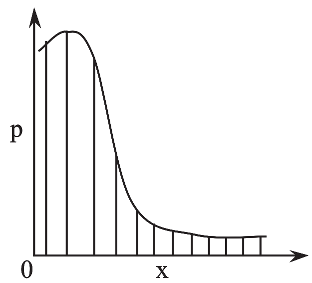


Figure (ii)

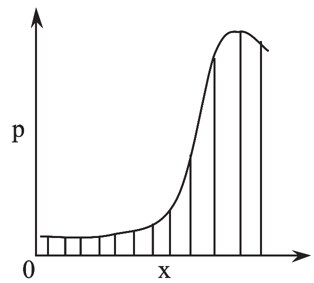


Figure (iii)

6. If 'n' is large and if neither 'p' nor 'q' is nearly zero, in such cases the binomial distribution is modified to normal distribution by standardizing the variable, $\boxed{Z = \frac{X - np}{\sqrt{npq}}}$

Mean of Binomial Distribution

The mean of binomial distribution is denoted by 'μ' or '$E(X)$' is the expected number of successes in 'n' number of trials.

$\therefore$ The mean of binomial distribution,

$$\boxed{\mu = np}$$

Where,

'n' = Number of trials

'p' = Probability of success in a single trial.

The two parameters of binomial distribution are 'n' and 'p'.

Variance of Binomial Distribution

The variance of binomial distribution is denoted by 'σ^2' and is the square of the standard deviation. Thus, the variance of binomial distribution is given by,

$$\boxed{\sigma^2 = npq}$$

Where,

n = Number of trials

p = Probability of success in a single trial

q = Probability of failure in a single trial.

Standard deviation is given by,

$$\boxed{\sigma = \sqrt{npq}}$$

$$\Rightarrow \boxed{\text{Standard deviation} = \sqrt{\text{Variance}}}$$

$$\Rightarrow \boxed{\text{Variance, } \sigma^2 = \mu . q} \quad [\because \mu = np]$$

5.1.1 Conditions and Constants Under Binomial Distribution

Q11. Write about the conditions and constants under which binomial distribution is used.

Answer :

Conditions Under which Binomial Distribution is Used

The following are the conditions which must be fulfilled for using the binomial distribution,

1. Each trial has only two mutually exclusive outcomes and collective exhaustive outcomes which are referred as "Success" and "Failure".
2. The outcomes of the trial 'success' is denoted by 'P' and failure denoted by 'q'.
3. The experiment is repeated under the same conditions for a fixed and finite number of times.
4. Each observation of random variable in an random experiment is known as trial.
5. The possibility of outcome of any trail does not change and remains constant compared to previous trials.
6. The trials are independent i.e., outcomes of one trial has no effect on the outcome of other.

Constants of Binomial Distribution

The random variable 'x' the number of successes 'r' in n trials has a probability distribution.

$P(x = r) = {}^nC_r \, p^r \, q^{n-r}$

Where,

$r = 1, 2, ... n$

p = Probability of successes on a single trial

q = Probability of failure on a single trial

n = Number of Bernoulli trials

Mean : $\mu = np$, Variance : $\sigma^2 = npq$

The following are the constants of binomial distribution,

(i) For binomial distribution variance is less than mean.

(ii) For binomial distribution with parameters n and p variance cannot exceed $n/4$.

(iii) Mean and variance of a binomial random variable depend on values assumed by parameters n, p and q.

The various constants of the binomial distribution can be listed in the following table,

Mean	$= np$
Standard Deviation	$= \sqrt{npq}$
Variance (μ_2)	$= npq$
Skewness (μ_3)	$= \dfrac{q-p}{\sqrt{npq}}$
Kurtosis (μ_4)	$= \dfrac{1-6pq}{npq}$

5.1.2 Fitting of Binomial Distribution

Q12. Briefly describe about fitting a binomial distribution along with an illustrati

Answer :

Fitting a Binomial Distribution

The following are the steps to be considered while fitting a binomial distribution,

Step-1

Calculate the values 'p' and 'q' if one is known other can be obtained as $p + q = 1$

Step-1

Expand binomial $(p + q)^n$

Where,

n = One less than the number of terms obtained after expansion.

For example, if 'n' value is 3, then number of terms obtained after expansion is 4.

Step-3

Multiply each term of the expanded binomial by 'N' which is the sum of all the frequencies to obtain the expected individual frequencies.

Illustration

For answer refer Unit-V, Page No. 130, Q.No. 13.

PROBLEMS ON BINOMIAL DISTRIBUTION

Q13. Fit a binomial distribution,

X	0	1	2	3	4	5	6	7
Y	7	6	19	35	30	23	7	1

Solution :

Steps in the Fitting of Binomial Distribution

Step 1

Calculate the values p and q

Mean of binomial distribution $= np$

Mean of frequency distribution $= \dfrac{\Sigma fx}{\Sigma f}$

Here 'X' is the random variable for success and the number of Bernoulli trials here is $n = 7$ (As there are 8 terms, 'n' value is always taken as one less than the number of terms).

Mean can be calculated by the given below,

x	f	f(x)
0	7	0
1	6	6
2	19	38
3	35	105
4	30	120
5	23	115
6	7	42
7	1	7
Total	**Σf = 128**	**Σf(x) = 433**

From the above table, $\Sigma f(x) = 433$

$\Sigma f = 128$

$$\text{Mean} = \frac{\Sigma f(x)}{\Sigma f}$$

$$= \frac{433}{128}$$

$$= 3.38$$

$\therefore$ Mean = 3.38 ... (1)

Mean = np ... (2)

[From mean of binomial distribution]

From equations (1) and (2) we get,

$\therefore \quad np = 3.38$

$[\because n = 7$ (from given problem)$]$

$\therefore \quad 7p = 3.38$

$$p = \frac{3.38}{7}$$

$$= 0.48$$

$\therefore$ The probability of success, $p = 0.48$

Probability of failure, $q = 1 - 0.48$

$[\because p + q = 1]$

$= 0.52$

$\therefore \quad p = 0.48$ and $q = 0.52$

Step 2

Expand Binomial $(p + q)^n$

The expected binomial probabilities are calculated by using the formula of Bernoulli distribution.

This is given by,

$$p(r) = {}^nC_r p^r q^{n-r}$$

$n = 7$ ($\because$ from table) $\quad p = 0.48$

$r = 0, 1, 2, 3, 4, 5, 6, 7$ (given) $\quad q = 0.52$

$p(r) = {}^nC_r . p^r q^{n-r}$

$p(0) = {}^7C_0(0.48)^0(0.52)^7 \quad [\because {}^nC_0 = 1]$

$= 1 \times 1 \times 0.01 = 0.01 \quad [x^0 = 1]$

$\therefore \quad p(0) = 0.01$

$p(1) = {}^7C_1(0.48)^1(0.52)^6 \quad [\because {}^nC_1 = n]$

$= 7 \times 0.48 \times 0.019$

$\therefore \quad p(1) = 0.064 \quad \left[{}^nC_r = \frac{n!}{r!(n-r)!}\right]$

$p(2) = {}^7C_2\ (0.48)^2\ (0.52)^5$

$= \frac{7 \times 6}{2} \times 0.23 \times 0.038$

$= 21 \times 0.23 \times 0.038$

$\therefore \quad p(2) = 0.18$

$p(3) = {}^7C_3(0.48)^3(0.52)^4$

$= \frac{7 \times 6 \times 5}{3 \times 2} \times 0.11 \times 0.073$

$= 35 \times 0.11 \times 0.073$

$\therefore \quad p(3) = 0.28$

$p(4) = {}^7C_4(0.48)^4(0.52)^3 \quad [{}^7C_3 = {}^7C_4 = 35]$

$= 35 \times 0.053 \times 0.14$

$\therefore \quad p(4) = 0.26$

$p(5) = {}^7C_5(0.48)^5(0.52)^2 \quad [{}^7C_5 = {}^7C_2 = 21]$

$= 21 \times 0.026 \times 0.27$

$\therefore \quad p(5) = 0.15$

$p(6) = {}^7C_6(0.48)^6(0.52)^1 \quad [{}^7C_6 = {}^7C_1 = 7]$

$= 7 \times 0.012 \times 0.52$

$\therefore \quad p(6) = 0.04$

$p(7) = {}^7C_7(0.48)^7(0.52)^0 \quad [\because {}^nC_n = 1]$

$= 1 \times 0.006 \times 1 \quad [x^0 = 1]$

$\therefore \quad p(7) = 0.006$

Step 3

Multiply each term with total frequency (N) to obtain expected frequencies

The expected frequencies of the distribution are given by,

$$f(r) = N.p(r)$$

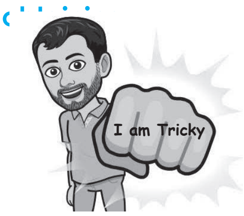
Filling of Binomial Distribution

r	$p(r) = {}^nC_r.p^rq^{n-r}$	f(r) = N.p(r) = 128.p(r)
0	$p(0) = {}^7C_0(0.48)^0(0.52)^7 = 0.01$	$f(0) = 128 \times 0.01 = 1.28$
1	$p(1) = {}^7C_1(0.48)^1(0.52)^6 = 0.064$	$f(1) = 128 \times 0.064 = 8.192$
2	$p(2) = {}^7C_2(0.48)^2(0.52)^5 = 0.18$	$f(2) = 128 \times 0.18 = 23.04$
3	$p(3) = {}^7C_3(0.48)^3(0.52)^4 = 0.28$	$f(3) = 128 \times 0.28 = 35.84$
4	$p(4) = {}^7C_4(0.48)^4(0.52)^3 = 0.26$	$f(4) = 128 \times 0.26 = 33.28$
5	$p(5) = {}^7C_5(0.48)^5(0.52)^2 = 0.15$	$f(5) = 128 \times 0.15 = 19.2$
6	$p(6) = {}^7C_6(0.48)^6(0.52)^1 = 0.04$	$f(6) = 128 \times 0.04 = 5.12$
7	$p(7) = {}^7C_7(0.48)^7(0.52)^0 = 0.006$	$f(7) = 128 \times 0.006 = 0.768$

Note

1. Always the sum of all the probabilities is equal to 1
2. Always the sum of all the calculated expected frequencies is equal to the sum of frequencies ($N = \Sigma f$) mentioned in the problem.

Q14. Ten unbiased coins are tossed simultaneously. Find the probability of



(i) Exactly 6 Heads

(ii) Atleast 8 Heads

(iii) No Heads

(iv) Atleast one Head

(v) Not more than 3 Heads and

(vi) Atleast 4 heads.

Solution : (Model Paper-II, Q13(b) | May/June-19, Q13(a) (OU))

According to Binomial Probability of law, Probability of r heads is given by,

$P(r) = P(X = r) = {}^nC_r p^r q^{n-r}$

Where, ${}^nC_r = \dfrac{n!}{r!(n-r)!}$

P = Probability of obtaining head = $\dfrac{1}{2}$

q = Probability of obtaining tail = $\dfrac{1}{2}$

$n = 10$

(i) Probability of Obtaining Exactly 6 Heads

$n = 10, r = 6, p = \dfrac{1}{2}, q = \dfrac{1}{2}$

$$P(6 \text{ heads}) = {}^{10}C_6 \left(\frac{1}{2}\right)^6 \left(\frac{1}{2}\right)^{10-6}$$

$$= {}^{10}C_6 \left(\frac{1}{2}\right)^6 \left(\frac{1}{2}\right)^4$$

$${}^{10}C_6 = \frac{10!}{6!(10-6)!} = \frac{10!}{6!(4!)}$$

$$= \frac{10 \times 9 \times 8 \times 7 \times 6!}{6!(4 \times 3 \times 2 \times 1)}$$

$$= 210$$

$\therefore \quad P(6 \text{ heads}) = 210\left(\frac{1}{64}\right)\left(\frac{1}{16}\right)$

$= \frac{210}{1,024}$

$= 0.205$

$\therefore$ Probability of obtaining exactly 6 heads is 0.205.

(ii) Probability of Obtaining Atleast Eight Heads

$P(X \geq 8) = P(8 \text{ heads}) + P(9 \text{ heads}) + P(10 \text{ heads})$

$$P(8 \text{ heads}) = {}^{10}C_8\left(\frac{1}{2}\right)^8\left(\frac{1}{2}\right)^{10-8}$$

$$= {}^{10}C_8\left(\frac{1}{2}\right)^8\left(\frac{1}{2}\right)^2$$

$$= \frac{10!}{8!(10-8!)}\left(\frac{1}{256}\right)\left(\frac{1}{4}\right)$$

$$= \frac{10\times9\times8!}{8!(2!)}\left(\frac{1}{256}\right)\left(\frac{1}{4}\right)$$

$$= 45\left(\frac{1}{256}\right)\left(\frac{1}{4}\right)$$

$$= \frac{45}{1024}$$

$$= 0.044$$

$$P(9 \text{ heads}) = {}^{10}C_9\left(\frac{1}{2}\right)^9\left(\frac{1}{2}\right)^{10-9}$$

$$= \frac{10!}{9!(10-9!)}\left(\frac{1}{512}\right)\left(\frac{1}{2}\right)$$

$$= \frac{10\times9!}{9!(1)}\left(\frac{1}{1024}\right)$$

$$= \frac{10}{1024}$$

$$= 0.01$$

$$P(10 \text{ heads}) = {}^{10}C_{10}\left(\frac{1}{2}\right)^{10}\left(\frac{1}{2}\right)^{10-10}$$

$$= 1\left(\frac{1}{1024}\right)(1)$$

$$= 0.001$$

Now, $P(X \geq 8) = 0.044 + 0.01 + 0.001$

$= 0.055$

$\therefore$ Probability of obtaining atleast 8 heads is 0.055.

(iii) Probability of Obtaining No Heads

$$P(0 \text{ heads}) = {}^{10}C_0\left(\frac{1}{2}\right)^0\left(\frac{1}{2}\right)^{10-0}$$

$$= 1\,(1)\left(\frac{1}{1024}\right)$$

$$= 0.001$$

$\therefore$ Probability of obtaining no heads is 0.001

(iv) Probability of Obtaining Atleast One Head

$P(\text{atleast one head}) = 1 - P(\text{no heads})$

[From (iii), $P(\text{no heads})$] = 0.001] $= 1 - 0.001$

$= 0.99$

$\therefore$ Probability of obtaining atleast one head is 0.001

(v) Probability of Obtaining Not More Than 3 Heads

$P(X \leq 3) = P(0 \text{ heads}) + P(1 \text{ heads}) + P(2 \text{ heads}) + P(3 \text{ heads})$

$P(0 \text{ heads}) = 0.001$

$$P(1 \text{ head}) = {}^{10}C_1\left(\frac{1}{2}\right)^1\left(\frac{1}{2}\right)^{10-1}$$

$$= 10\left(\frac{1}{2}\right)\left(\frac{1}{512}\right) = \frac{10}{1024}$$

$$= 0.01$$

$$P(2 \text{ heads}) = {}^{10}C_2\left(\frac{1}{2}\right)^2\left(\frac{1}{2}\right)^{10-2}$$

$$= \frac{10!}{8!\times2!}\left(\frac{1}{4}\right)\left(\frac{1}{256}\right)$$

$$= \frac{10\times9\times8!}{8!\times2\times1}\left(\frac{1}{1024}\right)$$

$$= 45\left(\frac{1}{1024}\right)$$

$$= 0.044$$

$$P(3 \text{ heads}) = {}^{10}C_3\left(\frac{1}{2}\right)^3\left(\frac{1}{2}\right)^{10-3}$$

$$= \frac{10!}{7!\times3!}\left(\frac{1}{8}\right)\left(\frac{1}{128}\right)$$

$$= \frac{10\times9\times8\times7!}{7!\times3\times2\times1}\left(\frac{1}{1024}\right)$$

$$= \left(\frac{120}{1024}\right)$$

$$= 0.12$$

$P(X \leq 3) = 0.001 + 0.01 + 0.044 + 0.12$

$= 0.175$

$\therefore$ Probability of obtaining not more than 3 heads is 0.175

(vi) Probability of Obtaining Atleast Four Heads

$P(X \geq 4) = 1 - P(X \leq 3)$

[From (v) $P(X \leq 3) = 0.175$]

Now, $P(X \geq 4) = 1 - 0.175$

$= 0.825$

$\therefore$ Probability of obtaining atleast 4 heads is 0.825.

Q15. Five coins are tossed 3,200 times, find the frequencies of the distribution of heads and tails; and tabulate the results and also calculate Mean and Standard Deviation of fitted distribution.

Solution : May/June-18, Q13(a) (OU)

In case of tossing a coin,

Probability of getting a head, $P = \frac{1}{2}$

Probability of not getting a head (tail), $q = \frac{1}{2}$

The expected frequencies are the successive terms of binomial expansion of $N(q + p)^n$

Where,

$n = 5, N = 3{,}200, P = \frac{1}{2}, q = \frac{1}{2}$

Binomial expansion of $N(q + p)^n$ i.e of $3{,}200 \left[\frac{1}{2} + \frac{1}{2}\right]^5$

$$= 3{,}200\left[\left[\frac{1}{2}\right]^5 + 5\left[\frac{1}{2}\right]^4\left[\frac{1}{2}\right] + 10\left[\frac{1}{3}\right]^3\left[\frac{1}{2}\right]^2 + 10\left[\frac{1}{2}\right]^2\left[\frac{1}{2}\right]^3 + 5\left[\frac{1}{2}\right]\left[\frac{1}{2}\right]^4 + \left[\frac{1}{2}\right]^5\right]$$

$$= 3{,}200\left[\left[\frac{1}{2}\right]^5 + 5\left[\frac{1}{2}\right]^5 + 10\left[\frac{1}{2}\right]^5 + 10\left[\frac{1}{2}\right]^5 + 5\left[\frac{1}{2}\right]^5 + \left[\frac{1}{2}\right]^5\right]$$

$$= 3{,}200 \times \left[\frac{1}{2}\right]^5 [1 + 5 + 10 + 10 + 5 + 1]$$

$= 100 [1 + 5 + 10 + 10 + 5 + 1]$

$= 100 + 500 + 1000 + 1000 + 500 + 100$

$= 3{,}200$

The frequencies of the distribution of heads and tails are tabulated below,

Number of Heads (x)	0	1	2	3	4	5
Frequency (F)	100	500	1,000	1,000	500	100

Calculation of Mean

Mean $= np$

$= 5 \times \frac{1}{2} = 2.5$

Calculation of Standard Deviation

$S.D(\sigma) = \sqrt{npq}$

$= \sqrt{5 \times \frac{1}{2} \times \frac{1}{2}} = \sqrt{\frac{5}{4}}$

$= 1.12$

Q16. In a city half of the population are Rice consumers to find this truth 100 supervisors are appointed. Every supervisor has been examined 10 members, what is the probability to three or less than three people are rice consumers reports supervisors number.

Solution : May/June-19, Q10(b) (MGU)

Note: In the given question, there is incomplete information due to which we have revised the question in order to solve it. The revised question (original question of text book) is as follows,

Suppose that half the population of a town are consumers of rice. 100 investigators are appointed to find out its truth. Each investigator interviews 10 individuals. How many investigators do you expect to report that three or less of the people interviewed are consumers of rice?

Let 'x' be the probability of persons out of 10 persons who are rice consumers.

From binomial distribution,

$$P(X=x) = {}^{n}C_{x}\ p^{x} q^{n-x}$$

Given that,

$$p = \frac{1}{2},\ q = 1-p = \frac{1}{2},\ n = 10$$

$$P(X=x) = {}^{10}C_{x}\left(\frac{1}{2}\right)^{x}\left(\frac{1}{2}\right)^{10-x}$$

∴ The probability that there are 3 people or less who are rice consumers.

$$p(X \le 3) = p(x=0) + p(x=1) + p(x=2) + p(x=3)$$

$$p(X=0) = {}^{10}C_{0}\left(\frac{1}{2}\right)^{0}\left(\frac{1}{2}\right)^{10-0} = 1\,(1)(1/2)^{10} = \frac{1}{2^{10}}$$

$$p(X=1) = {}^{10}C_{1}\left(\frac{1}{2}\right)^{1}\left(\frac{1}{2}\right)^{10-1} = 10\,(1/2)(1/2)^{9} = \frac{10}{2^{10}}$$

$$p(X=2) = {}^{10}C_{2}\left(\frac{1}{2}\right)^{2}\left(\frac{1}{2}\right)^{10-2} = 45\,(1/2)^{2}\,(1/2)^{8} = \frac{45}{2^{10}}$$

$$p(X=3) = {}^{10}C_{3}\left(\frac{1}{2}\right)^{3}\left(\frac{1}{2}\right)^{10-3} = 120\,(1/2)^{3}\,(1/2)^{7} = \frac{120}{2^{10}}$$

$$p(X \le 3) = \frac{1}{2^{10}} + \frac{10}{2^{10}} + \frac{45}{2^{10}} + \frac{120}{2^{10}}$$

$$= \frac{1+10+45+120}{2^{10}} = \frac{176}{1024} = \frac{11}{64}$$

$$= 0.171875$$

$$\simeq 0.172$$

∴ The expected number of supervisors who would report that there are '3' or less people were rice consumers.

$$= 100 \times 0.172 = 17.2$$

$$\simeq 17.$$

5.2 POISSON DISTRIBUTION – IMPORTANCE, PROPERTIES AND APPLICATIONS

Q17. Define poisson distribution and state its importance.

Answer :

Poisson Distribution

Poisson distribution is a discrete probability distribution for countably infinite trials. Poisson distribution is named after French mathematician, 'SIMEON DENIS POISSON' in 1837. It is used when the probability of success of any individual event is very small.

The average or mean of poisson distribution is given by λ. However, the single parameter of poisson distribution is also given as λ.

Poisson distribution can be used generally to approximate the binomial distribution when 'n' value is large (i.e., $n \to \infty$) and 'p' value is very small (i.e., $p \to o$)

Always the sum of infinite probabilites in poisson distribution is 1 i.e.,

$$\boxed{P(0)+P(1)+P(2)+\ldots+P(\infty)=1}$$

Importance of Poisson Distribution

Following points highlight the importance of Poisson Distribution,

1. It helps to describe the complete randomness and independence of events.
2. It helps to make unlimited number of trials.
3. It helps to find out the probability of events occurring in a fixed interval of time.

4. It is more effective and helpful compare to binomial distribution because it requires only Mean (M) whereas binomial distribution requires *n* and *p*.
5. It helps to find out the optimal size of a unit.
6. It helps in understanding the problems and finding their solutions.
7. It helps events with low probabilities of occurrence within some definite time or space.
8. It helps businessmen to make forecasts for number of customers or sales on certain days or seasons.

Q18. Explain about the properties and applications of poisson distribution.

OR

Explain the features of Poisson distribution. May/June-19, Q10(a) (MGU)

(Refer Only Topic: Properties/Features of Poisson Distribution)

Answer :

Properties/Features of Poisson Distribution

Following are the properties/features of Poisson distribution,

1. The occurrence of the events is independent i.e., the occurrence of an event in a time interval has no effect on the occurrence of the second event in the same or any other interval.
2. Theoretically, an infinite number of occurrences of the event must be possible in the interval.
3. The probability of single occurrence of the event in a given interval is directly proportional to the length of the interval.
4. In any extremely small portion of the interval, the probability of two or more occurrences of the event is negligible.

   Like binomial distribution, poisson distribution also satisfies the two essential properties i.e.,

   (i) $f(x) \geq 0$ and

   (ii) $\Sigma f(x) = 1$.
5. It is a discrete probability distribution.
6. It is positively skewed to right.

Applications of Poisson Distribution

Poisson distribution is mostly applied in business, management science and operations research. Some of the examples of applications which are observed in our daily life where poisson distribution is used are as follows,

(i) Number of calls received at a call centre.

(ii) Number of printing mistakes occurred on the pages in a book.

(iii) Number of trains arrived at railway station.

(iv) Number of persons joining a queue at a bank.

(v) Number of bacteria in a given media.

(vi) Number of typing mistakes detected by proofing department.

(vii) Number of particles emitted by a radio active substance.

(viii) Number of defective items identified in box containing very large number of items.

(ix) Number of customers served at a telephone department.

(x) Number of deaths in a village by an unknown disease.

5.2.1 Conditions and Constants Under Poisson Distribution

Q19. Write about the conditions and constants under which Poisson Distribution is used.

Answer :

Conditions Under Which Poisson Distribution is Used

Poisson distribution is a limiting case of binomial distribution when,

1. $n \rightarrow \infty$ i.e., number of trials is very large.
2. $P \rightarrow 0$ i.e., probability of success for each trial is very small.
3. $np = \lambda$ is a finite constant (positive real number)

Thus, $P = \frac{\lambda}{n}$ and $q = \left(1 - \frac{\lambda}{n}\right)$

Probability of 'x' success in a series of 'n' independent trials is,

$$\boxed{P(x) = \frac{e^{-\lambda}.\lambda^x}{x!}}$$

Where,

$x = 0, 1, 2, ..., n$

λ = Mean of poisson distribution.

Constants or General Formula of Poisson Distribution

The probability of 'X' occurrences in poisson distribution is given by,

$$\boxed{P(X) = \frac{e^{-\lambda}.\lambda^x}{x!}}$$

Where,

x = Random variable.

It may take 0, 1, 2, 3, ..., ∞

$e = 2.7183$

(The base of natural logarithms)

λ = Mean of poisson distribution

(Average number of occurrences of an event)

Mean can be calculated by multiplying 'n' and 'p'

$$\boxed{\lambda = np}$$

'λ' is the single parameter of Poisson distribution. As 'λ' increases, the distribution shifts to right.

Hence, it is called as 'Probability distribution of rare events' or "Law of improbable events".

It is a discrete probability distribution with single parameter 'λ'.

As the value of 'λ' increases the distribution shifts to the right. All poisson probability distribution are skewed to right.

It is a distribution of rare events. Here the finite number of trials i.e., the value of 'n' is not mentioned.

'n' tends to ∞ in this case.

Under Poisson distribution,

$$\boxed{\text{Mean} = \text{Variance} = \lambda}$$

5.2.2 Fitting of Poisson Distribution

Q20. Explain the steps of fitting a Poisson Distribution.

Answer :

The following are the steps for fitting a Poisson distribution,

Step 1

Calculate the values of mean (λ) and probability of zero occurrence.

Mean in poisson distribution is calculated as,

$$\boxed{\lambda = nxp}$$

Where,

n = Number of trials (very large)

p = Probability of successes (very small).

Step 2

Calculate the probabilities by using recurrence relation which is given below.

$$P(x) = \frac{e^{-\lambda}.\lambda^x}{x!} \Rightarrow P(0) = \frac{e^{-\lambda}.\lambda^0}{0!} = e^{-\lambda}$$

Always probability of zero occurrence,

$$P(0) = e^{-\lambda}$$

$$P(1) = \frac{P(0).\lambda}{1} = e^{-\lambda}.\lambda$$

$$P(2) = \frac{P(1).\lambda}{2}$$

$$P(3) = \frac{P(2).\lambda}{3} \text{ and so on.}$$

Step 3

Multiply each term of probabilities with total frequency (N) to obtain expected frequency values.

PROBLEMS ON POISSON DISTRIBUTION

Q21. Fit a Poisson distribution to the following data:

X	0	1	2	3	4
Y	211	90	19	5	0

(e^{-m} = 0.6443)

Solution : (Model Paper-I, Q13(b) | May/June-19, Q13(b) (OU))

Steps in the Fitting of Poisson Distribution

Step-1

Calculate the values of 'λ' and probability of zero occurrence

x	f	fx
0	211	0
1	90	90
2	19	38
3	5	15
4	0	0
	Σf = 325	Σfx = 143

Mean of Poisson distribution is given by $\lambda = \dfrac{\Sigma fx}{\Sigma f}$

$$\lambda = \frac{143}{325} = 0.44$$

$$\boxed{\therefore \lambda = 0.44}$$

Step-2

Calculate all the probabilies by using recurrence relation,

$p(0) = 0.6443 \qquad (\because \text{ Given } e^{-m} \text{ or } e^{-\lambda}\ 0.6443)$

$$p(1) = \frac{p(0)\times\lambda}{1}\ 0.6443 \times 0.44 = 0.2835$$

$$p(2) = \frac{p(1)\times\lambda}{2} = \frac{0.2835\times 0.44}{2} = 0.06237$$

$$p(3) = \frac{p(2)\times\lambda}{3} = \frac{0.06237\times 0.44}{3} = 0.00915$$

$$p(4) = \frac{p(3)\times\lambda}{4} = \frac{0.00915\times 0.44}{4} = 0.004$$

Step-3

Multiply each term of probability with total frequency (Σf) to obtain the values of expected frequencies

Computation of Expected Frequencies

x	P(X)	f(x) = N.P(X) = 325. P(x)
0	$P(0) = 0.643$	$f(0) = 325 \times 0.6443 = 209.3 \cong 209$
1	$P(1) = 0.2835$	$f(1) = 325 \times 0.2835 = 92.1375 \cong 92$
2	$P(2) = 0.06237$	$f(2) = 325 \times 0.06237 = 20.27 \cong 20$
3	$P(3) = 0.00915$	$f(3) = 325 \times 0.00915 = 2.974 \cong 3$
4	$P(4) = 0.004$	$f(4) = 325 \times 0.004 = 1.3 \cong 1$
	Total	**325**

$\therefore$ The theoretically fitted Poisson distribution is as follows,

X	0	1	2	3	4
Y	209	92	20	3	1

Q22. Six coins are tossed 6400 times. Find the probability to get 6 heads in 2 tosses using Poisson distribution.

I am Simple and Easy

Solution : (Model Paper-III, Q13(a) | May/June-18, Q6(b) (KU))

Probability of getting one head with one coin $= \dfrac{1}{2}$

$\therefore$ The probability of getting six heads with six coins $= \left(\dfrac{1}{2}\right)^6 = \dfrac{1}{64}$

The Average number of six heads with six coins in 6,400 throws, $m = np = 6{,}400 \times \dfrac{1}{64} = 100$

Approximately probability of getting six heads 2 times when the distribution is Poisson,

$$P(x=2) = \frac{e^{-m}.m^2}{2!} = \frac{e^{-100}.100^2}{(100)!}$$

Q23. Fit a Poisson Distributions for the following data by using direct method.

No of Deaths	0	1	2	3	4
Frequency	122	60	15	2	1

Solution :

We know that,

The probability mass function of poisson distribution is given by,

$$P(X = x) = \frac{e^{-\lambda}\lambda^{x}}{x!} \qquad \text{Where } x = 0, 1, 2, \ldots$$

Now, compute the mean and variance of poisson distribution i.e., λ0

x	f	fx
0	122	0
1	60	60
2	15	30
3	2	6
4	1	4
Total	**200**	**100**

Now, compute the expected frequencies,

x	$P(X = x) = \frac{e^{-\lambda}\lambda^{x}}{x!}$	Expected frequency N = P(X = x)
0	$\frac{e^{-0.5} \times (0.5)^0}{0!} = 0.6065$	200 (0.6065) = 121.3 ≅ 121
1	$\frac{e^{-0.5} \times (0.5)^1}{1!} = 0.3033$	200 (0.3033) = 60.66 ≅ 61
2	$\frac{e^{-0.5} \times (0.5)^2}{2!} = 0.0758$	200 (0.0758) = 15.16 ≅ 15
3	$\frac{e^{-0.5} \times (0.5)^3}{3!} = 0.0126$	200 (0.0126) = 2.52 ≅ 3
4	$\frac{e^{-0.5} \times (0.5)^4}{4!} = 0.0016$	200 (0.0016) = 0.32 ≅ 0

5.3 NORMAL DISTRIBUTION – IMPORTANCE

Q24. What is Normal Distribution? Write the importance and applications of normal distribution.

Answer :

Normal Distribution

Normal distribution was first discovered by 'Abraham Demoivre' in 1733 as a limiting case of binomial distribution. It was later developed by LAPLACE and GAUSS. It is also known as 'Gaussian distribution' as the credit goes to German mathematician 'Karl Friedrich Gauss'.

In normal distribution, the probability of occurrence of values of random variables are calculated within a range or interval whereas the probability of a particular value cannot be calculated and is always assumed to be zero. The probability distribution used mostly for dealing with quantities whose magnitude is continuously variable is normal distribution. Hence, it is called as 'Normal probability distribution'. It plays a prominent role in statistical theory and practice.

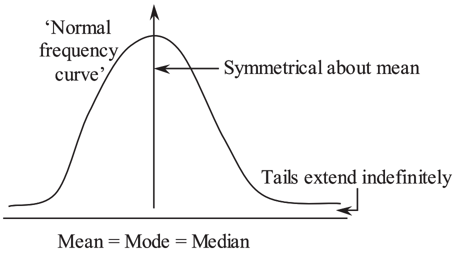
It is an approximation to binomial distribution whether p is equal to q or not. Binomial distribution tends to form a continuous curve when 'n' becomes large at least for a certain range.

Thus, the limiting frequency curve which is obtained when '*n*' has large value is called as 'the normal frequency curve' or 'the normal curve'.

The two parameters of normal distribution are 'μ' (mean) and 'σ^2' (variance).

Always the value of random variable '*X*' lies within certain range and has no particular value.

Thus, $-\infty < X < \infty$



Importance of Normal Distribution

Following points highlights the importance of normal distribution,

1. Many of the distributions like Binomial, Poisson etc., can be approximated by normal distribution under suitable conditions.
2. It is used to fit a distribution under certain conditions.
3. Many of the distributions of sample statistics i.e., the distribution of sample mean, sample variance etc., tend to normality for large samples.
4. It is used for large applications in statistical quality control in industry for establishing control limits.
5. It is widely or extensively used in the entire theory of small sample tests in which it is assumed that the parent population from which the samples have been drawn follow normal distribution.
6. It plays a major role in appropriate decision making under sampling theory.
7. Most of the sampling distributions of statistics such as, students t-distribution, Snedecor's F-distribution, Fisher's Z-distribution and Chi square distribution; adjust into normal distributions for large degrees of freedom.

Applications of Normal Distribution

Normal distribution plays a prominent role in statistical theory as well as practical area of applications. Some of the applications are,

1. It is used in astronomical observations for analyzing the measurement errors.
2. It is used to determine the blood pressure of human body.
3. It is used to determine the height of an individual.

Q25. Write about,

(i) General model of normal distribution

(ii) Standard normal distribution.

Answer :

(i) General Model of Normal Distribution

Under normal distribution, the random variable '*X*' is given as follows,

$$F(X) = \frac{1}{\sigma\sqrt{2\pi}} e^{\frac{-1}{2}\left(\frac{X-\mu}{\sigma}\right)^2}$$

Where,

X = Values of the continuous random variable $-\infty < X < \infty$

μ = Mean of the random variable

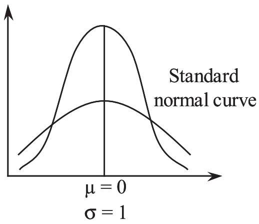
e = Mathematical constant (e = 2.7183)

π = Mathematical constant ($\pi = 3.14$) ($2\pi = 2.5066$)

σ = Standard deviation.

Standard normal curve is a curve in which whose $\mu = 0$ and $\sigma = 1$

Always it is known that the normal curve has unit area, hence, $\sigma = 1$



The graph of $f(X)$ is a famous 'bell-shaped curve'.

As the curve has unit area, the total frequency (N) is equal to 1.

It is not possible to draw normal curves in all cases, hence we standardize the normal curve is known as 'standard normal curve'.

Thus, '*X*' is converted into '*Z*' known as '*standard normal variate*'.

Standard normal variate '*Z*' is given by,

$$z = \frac{X-\mu}{\sigma}$$

Where,

X = Value of the observation

μ = Mean

σ = Standard deviation to F(Z).

$F(X)$ is also changed to $F(Z)$

$$F(Z) = \frac{1}{\sqrt{2\pi}} e^{\frac{-Z^2}{2}}$$

(ii) Standard Normal Distribution

The distribution of a normal random variable with mean (μ) = 0 and standard deviation (σ) = 1 is called as 'standard normal distribution' and the curve is called as 'standard normal curve'.

The random variable 'x' is said to have a normal distribution with the two parameters 'μ' and 'σ' if its probability function is given by,

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{\frac{-1}{2}\left(\frac{x-\mu}{\sigma}\right)^2} \qquad -\infty < x < \infty$$

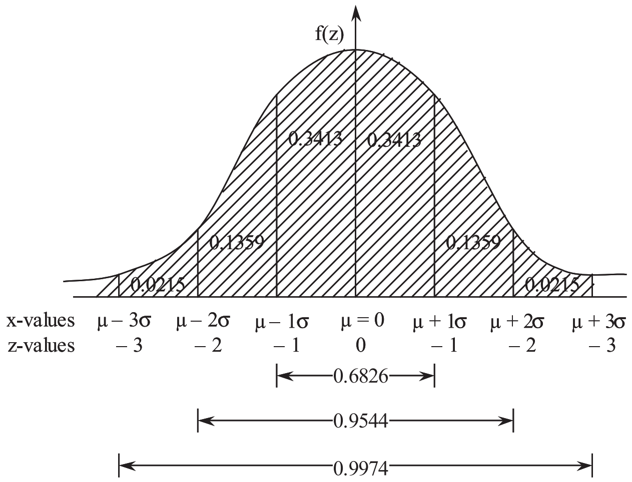
These two parameters describe the complete situation. The probability function has maximum value at its mean and decreases gradually on either side.

Sometimes, it is not possible to draw the normal curve, hence we standardize the normal curve known as 'standard normal curve'.

Thus 'x' is converted into 'z' and is known as 'standard normal variate'.

$$\therefore z = \frac{x-\mu}{\sigma}$$

Thus, in order to simplify the calculations standard normal variate 'z' is derived. It is possible to convert normal distribution to the standardized form because it has same shape whatever may be the values of parameters (μ and σ)



Figure

5.3.1 Central Limit Theorem

Q26. Explain in detail about central limit theorem.

Answer :

Central Limit Theorem

Central Limit Theorem states that the distribution of the sum of I.I.D (Independently and Identically distributed) random variables will be normal asymptotically under general conditions with mean $\mu = \sum_{i=1}^{n} \mu_i$ and standard deviation σ where $\left(\sigma^2 = \sum_{i=1}^{n} \sigma_1^2\right)$.

Therefore the distribution $S_n = X_1 + X_2 + X_3 + \ldots\ldots X_n$ i.e., $S_n = X_i$ where S_n is the random variable whose mean $\mu_i = E(X_i)$ and variance $\sigma_1^2 = V(X_i)$.

Following are some of the cases of Central Limit Theorem (C.L.T).

1. De-Moivre's Laplace Theorem

It is the first case of central limit theorem which was stated by Laplace. According to this theorem, the distribution of random variables with respect to the probability of success (P) is asymptotically normal as n tends to infinity.

It can be written as if a random variable,

$$X_i = \begin{cases} 1 & \text{if probability is } p \\ 0 & \text{if probability is } q \end{cases} \quad \text{where } i = 1, 2, 3, \ldots . n$$

Then the distribution $Sn = X_1 + X_2 + X_3 \ldots . Xn$ is normal as $n \to \infty$.

2. Lindeberg-Levy Theorem

This theorem was proposed by Lindeberg and Levy by considering two assumptions.

(i) The distribution of random variables is independent and identical.

(ii) Variance (σ^2) must be finite.

This theorem states that under the above assumptions if the random variables are distributed with $E(X_i) = \mu_1$ and $V(X_i) = \sigma^2$ then the sum $S_n = X_1 + X_2 + X_3 + \ldots . + X_n$ follows normal distribution where $\mu = n\mu_1$ (mean) and $\sigma^2 = n\sigma_1^2$ (variance).

3. Liapounoff's Central Limit Theorem

This is a generalized case of central limit theorem where the distribution of random variables is not identical. In this case third absolute moment (ρ^3) is considered whose distribution can be given as,

$$\rho^3 = \sum_{i=1}^{n} \rho_i^3$$

Then under general conditions, if $E(X_i) = \mu_i$ and $V(X_i) = \sigma_1^2$ and $\underset{n\to\infty}{Lt} = \frac{p}{\sigma} = 0$, then the sum $X = X_1 + X_2 + X_3$ $\ldots . . X_n$ follows normal distribution at $N(\mu_1, \sigma^2)$ with mean $\mu = \sum_{i=1}^{n} \mu_i$ and variance $\sigma^2 = \sum_{i=1}^{n} \sigma_i^2$

PROBLEM ON CENTRAL LIMIT THEOREM

Q27. Let $X_1, X_2, \ldots$ be a I.I.D. Poisson variates with parameter λ. Use CLT to estimate $P(120 \le S_n \le 160)$, where $S_n = X_1 + X_2 + X_3 + \ldots . + X_n$, $\lambda = 2$ and $n = 75$.

Solution :

Given that,

$X_1, X_2, X_3, \ldots ., X_n$ I.I.D Poisson variates with parameter λ.

$\lambda = 2$

$n = 75$

Since given X_i's are IDD of poisson variate we have,

$E(X_i) = \lambda$ and

$\text{var}(X_i) = \lambda$

Where, $i = 1, 2, 3, \ldots . . n$

Therefore,

$$E(S_n) = \sum_{i=1}^{n} E(X_i)$$

$$= n.E(X_i)$$

$$= n\lambda \qquad \ldots (1) \quad (\because E(X_i) = X)$$

Variance $(S_n) = \text{Var}(X_1 + X_2 + X_3 \ldots\ldots X_n)$

$$= \sum_{i=1}^{n} Var\ X_i$$

$= n\ \text{Var}(X_i)$

$= n\,\lambda$ ... (2) $(\because\ \text{Var}(X_i) = V)$

Since n is large (i.e., $n = 75$) then by Lindeberg-Levy CLT we have,

$S_n \sim N(n\,\lambda, n\,\lambda)$

$N(n\,\lambda, n\,\lambda)$

$= N(75 \times 2, 75 \times 2)$ $(\because\ n = 75, \lambda = 2)$

$= N(150, 150)$

$S_n = N(\mu = 150, \sigma^2 = 150)$ (μ = Mean and σ^2 = Variance)

$\therefore$ $P(120 < S_n < 160)$ is,

$$P(120 < S_n < 160) = P\left[\frac{120-\mu}{\sqrt{\sigma}} \le Z \le \frac{160-\mu}{\sqrt{\sigma}}\right]$$

$$= P\left[\frac{120-150}{\sqrt{150}} \le Z \le \frac{160-150}{\sqrt{150}}\right] = P\left[\frac{-30}{12.24} \le Z \le \frac{10}{12.24}\right]$$

$= P[-2.451 \le Z \le 0.816]$

$= P[-2.451 \le Z \le 0] + P[0 \le Z \le 0.816]$

From the normal distribution table the value of $P[-2.451 \le Z \le 0] = 0.4929$ and $P[0 \le Z \le 0.816] = 0.2939$.

$= 0.4929 + 0.2939$

$= 0.7868$

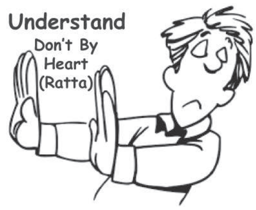
$\therefore$ $P[120 < S_n < 160] = 0.7868$.

5.3.2 Characteristics of Normal Distribution

Q28. What is normal distribution? State its characteristics.

OR

What is normal distribution? Write its any five features.



Answer : (Model Paper-II, Q13(a) | May/June-18, Q6(a) (KU))

Normal Distribution

In normal distribution, the probability of occurrence of values of random variables are calculated within a range or interval whereas the probability of a particular value cannot be calculated and is always assumed to be zero. The probability distribution used mostly for dealing with quantities whose magnitude is continuously variable is normal distribution. Hence, it is called as 'Normal probability distribution'. It plays a prominent role in statistical theory and practice.

Characteristics/Properties of Normal Distribution

Following are the characteristics/features/properties of normal distribution,

1. The normal curve is 'bell-shaped' and symmetrical about the mean (skewness = 0). If the curve is folded along its central vertical axis the curves either side of the axis would coincide.
2. The height of the normal curve is maximum at its mean. Hence, the mean and mode coincide. Thus, in normal distribution, mean, mode and median are equal.
3. The height of the curve is maximum at its mean but reduces as it goes towards either of the direction but never touches the base. Hence, the curve is known as ASYMPTOTIC. The range in unlimited or infinite in both the directions.

4. As there is only one maximum point, the normal curve has only one mode and it known as 'unimodal'.
5. The points of inflexion i.e., the points where the change in curvature occurs are $\bar{x} \pm \sigma$ (or) $\mu \pm \sigma$.
6. The variables used in binomial and Poisson are discrete variables whereas normal distribution has continuous random variable.
7. The first and third quartiles are at same distance from the median.
8. The area under the normal curve is distributed as follows,
   (i) Mean ± 1 σ covers 68.27% area
   (ii) Mean ± 2 σ covers 95.45% area
   (iii) Mean ± 3 σ covers 99.73% area
9. Mean of normal distribution may be negative, zero or positive.
10. The total area under the normal curve for normal probability distribution is 1.

5.3.3 Fitting A Normal Distribution (Areas Method Only)

Q29. Write about the area under the normal curve and normal distribution.

Answer :

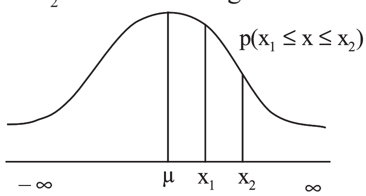
Area under any normal curve is found from the table of standard normal probability distribution showing the area between the mean and any value of the normally distributed random variable.

Since for different values of μ and σ we have different normal curves. Hence it is not possible to draw the normal curves for various values of μ and σ. Thus, the normal curve is transformed into a standardized normal curve.

'x' is transformed into 'z' which is known as 'standard normal variate'.

$$\therefore \quad \boxed{f(z) = \frac{1}{\sqrt{2\pi}} e^{\frac{-z^2}{2}}}$$

The area under the normal curve between the ordinates x = 'x_1' and x = 'x_2' gives the probability that the normal variate lies between 'x_1' and 'x_2' as shown in figure.



Figure

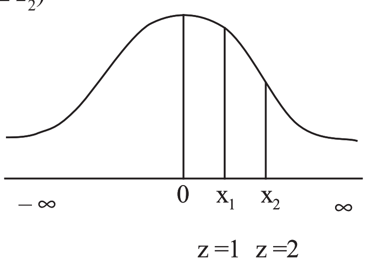
The probability can be evaluated as standardized 'z' as follows,

Substitute, $z = \dfrac{x-\mu}{\sigma}$ $\quad [\mu = 0, \ \sigma = 1]$

When, $x = x_1$, $z = \dfrac{x_1 - \mu}{\sigma} = z_1$ (suppose)

When, $x = x_2$, $z = \dfrac{x_2 - \mu}{\sigma} = z_2$

$\therefore \quad p(x_1 \le x \le x_2) = p(z_1 \le z \le z_2)$

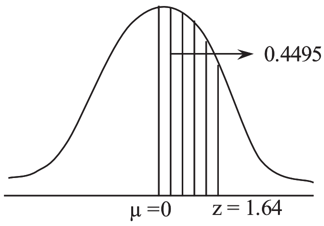


Figure

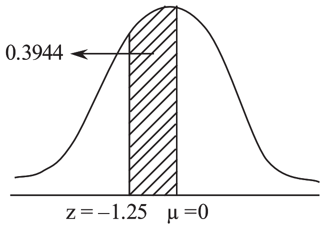
Examples

1. The area under the normal curve for $z = 1.64$

2. The area under the normal curve for $z = -1.25$



Figure



Figure

From normal distribution table, area values are taken and shade the region obtained.

If $z = 1.64$, Area = 0.4495

$z = 1.25$, Area = 0.3944

Area Property of the Normal Distribution

The area of the standard normal curve under normal distribution is equal to one. The curve approaches horizontal axis but never touches it, hence the curve is said to be 'asymptotic'.

The area under the normal curve is split into two halves by the mean $\mu = 0$ the area on either side of the mean is 0.5.

PROBLEMS ON NORMAL DISTRIBUTION

Q30. A study of past participants indicates that the mean length of time spent on the programme is 500 hours; and that, this normal distribution random variable has a standard deviation of 100 hours. What is the probability that a participant selected at random will required to complete the programme in following cases:

(i) 'More' than 500 hrs

(ii) Between 500 and 650 hrs

(iii) Between 550 and 650 hrs

(iv) Less than 580 hrs

(v) Between 420 and 570 hrs.

I am complex but Worthy

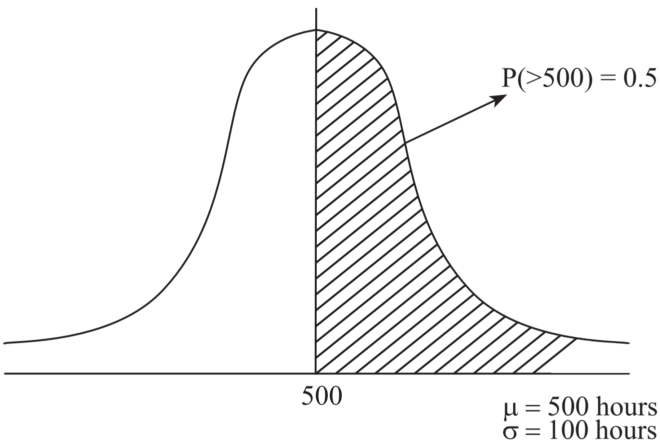
Solution : (Model Paper-III, Q13(b) | May/June-18, Q13(b) (OU))

Let, 'X' be length of time spent on programme (in hours) that follows normal duration.

Mean length of time spent on programme, $\mu = 500$ hours

Standard deviation = 100 hours.

(i) More than 500 hours

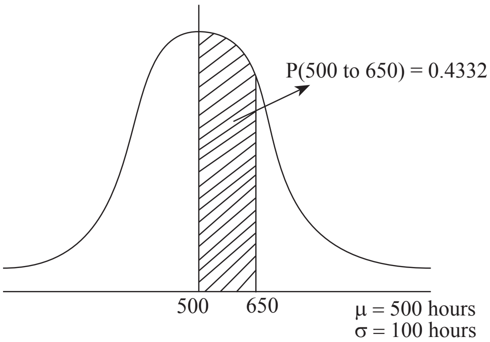


Curve's half of the area is located on either side of the mean of 500 hours. Therefore, it can be ascertain for the probability that a random variable would take on a value greater than 500 is the half shaded or 0.5.

(ii) Between 500 and 650 Hours

$P(500 \text{ to } 650)$

$$Z = \frac{X - \mu}{\sigma} = \frac{650 - 500}{100} = \frac{150}{100}$$
$$= 1.5$$



From the normal distribution table, the probability for $z = 1.5$ is 0.4322. Therefore, the probability that a candidate selected at random will require between 500 hours and 650 hours to complete the training program is 0.4322.

(iii) Between 550 and 650 Hours

$P(550 \text{ to } 650)$

When , $x = 550$

$$\text{Corresponding Z value, } Z = \frac{550 - 500}{100}$$
$$= \frac{50}{100}$$
$$= 0.5$$

When, $x = 650$

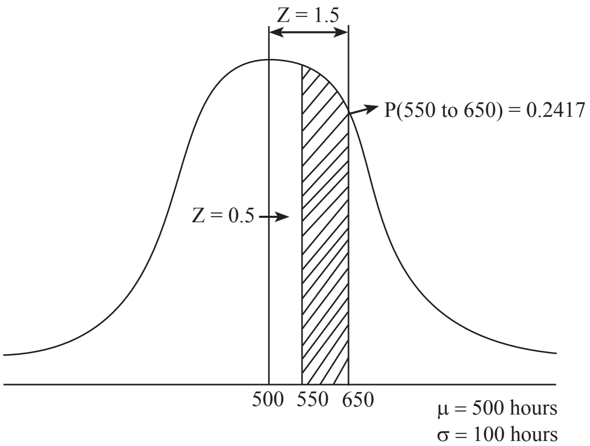
$$Z = 650 - 500 = \frac{150}{100}$$
$$= 1.5$$

The area between $Z = 0.5$ and $Z = 1.5$ i.e the shaded region in the graph is shown below,

From normal distribution table,

Area at $Z = 1.5$ is 0.4332

Area at $Z = 0.5$ is 0.1915



∴ The required area = 0.4332 – 0.1915 = 0.2417

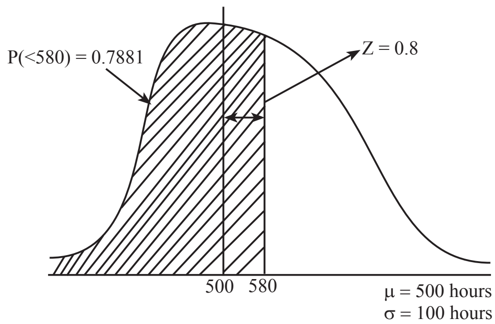
∴ The probability that a candidate selected at random will take between 550 hours and 650 hours to complete the training program is 0.2417.

(iv) Less than 580 Hours

$P(< 580)$

When,

$$x = 580, \text{ the corresponding } Z = \frac{580 - 500}{100}$$
$$= \frac{80}{100}$$
$$= 0.8$$



From normal distribution table,

Area at Z = 0.8 = 0.2881

The required area = 0.2881 + 0.5

= 0.7881

∴ The probability that a candidate selected at random will take lesser than 580 hours to complete the program is 0.7881.

(v) Between 420 and 570 Hours

P(420 to 570)

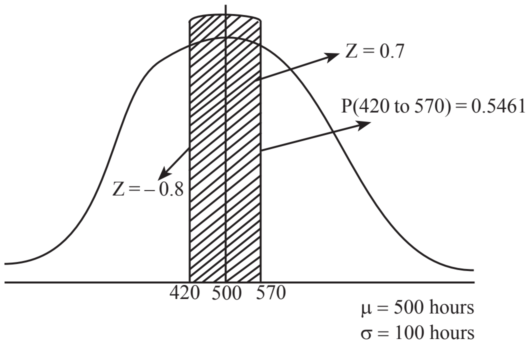
When $x = 420$

$$\text{Corresponding Z value, Z} = \frac{420 - 500}{100}$$
$$= \frac{-80}{100}$$
$$= -0.8$$

When , $x = 570$

$$Z = \frac{570 - 500}{100}$$
$$= \frac{70}{100}$$
$$= 0.7$$

The area between Z = – 0.8 and 0.7 i.e,. the shaded region in the graph given below,



From normal distribution table,

Area at $Z = -0.8 = 0.2881$

Area at $Z = 0.7 = 0.2580$

∴ The required area = 0.2881 + 0.2580 = 0.5461

∴ The probability that a candidate selected at random will take between 420 hours and 570 hours to complete the training program is 0.5461.

Q31. X is normally distributed and mean and s.d of x is 12 and 4. Find out the following probabilities,

(i) X ≥ 20

(ii) X ≤ 20

(iii) 0 ≤ X ≤ 12.

Solution :

Given that,

Mean, $\mu = 12$

Standard deviation, $\sigma = 4$

If $X \sim N(\mu, \sigma^2)$ then $Z = \dfrac{X-\mu}{\sigma}$

(i) $P(X \geq 20) = P\left(Z \geq \dfrac{X-\mu}{\sigma}\right)$

$= P\left(Z \geq \dfrac{20-12}{4}\right) = P\left(Z \geq \dfrac{8}{4}\right)$

$= P(Z \geq 2)$

$= 1 - P(Z < 2)$

$= 1 - [P(-\infty \leq Z \leq 0) + P(0 \leq Z \leq 2)]$

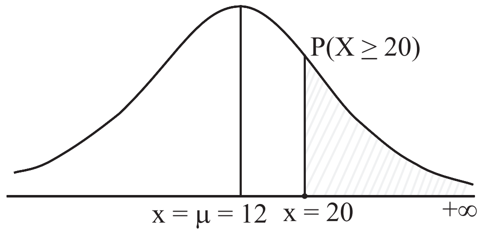
$= 1 - (0.5 + 0.4772)$

[∵ from table of normal distribution]

$= 1 - 0.97720$

$= 0.02280$

$\therefore P(X \geq 20) = 0.02280$



(ii) $P(X \leq 20) = P\left(Z \leq \dfrac{X-\mu}{\sigma}\right)$

$= P\left(Z \leq \dfrac{20-12}{4}\right) = P\left(Z \leq \dfrac{8}{4}\right)$

$= P(Z \leq 2)$

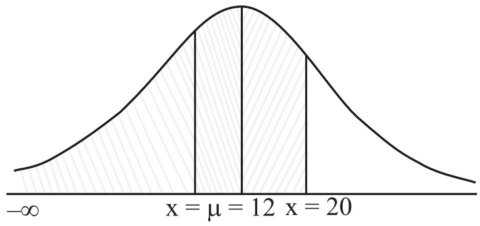
$= P(Z < 0) + P(0 < Z < 2)$

$= 0.5 + 0.4772$

[∵ from table of normal distribution]

$= 0.97720$

$\therefore P(X \leq 20) = 0.97720$



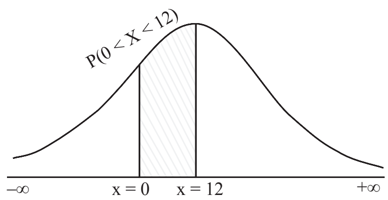
(iii) $P(0 \leq X \leq 12)$

When $X = 0$,

$$Z = \frac{X-\mu}{\sigma} = \frac{0-12}{4} = \frac{-12}{4} = -3$$

When $X = 12$,

$$Z = \frac{X-\mu}{\sigma} = \frac{12-12}{4} = \frac{0}{4} = 0$$



$P(0 \leq X \leq 12) = P(-3 < Z < 0) = P(0 < Z < 3)$

[∵ from table of normal distribution]

$= 0.4986$

Q32. The weekly wages of 1000 workmen are normally distributed, whose mean is ₹ 800 and the standard deviation is ₹ 50. Estimate the number of workers whose weekly wages will be,

(i) Between ₹ 800 and ₹ 900

(ii) Less than ₹ 750

(iii) Between ₹ 700 and ₹ 750 and

(iv) More than ₹ 900.

Solution :

Let, 'X' be the weekly wages of 1000 workmen. Wages are normally distributed. Mean of normal distribution,

$$\mu = ₹\ 800$$

Standard deviation, $\sigma = ₹\ 50$

(i) Between ₹ 800 and ₹ 900

The number of workers whose weekly wages will be ₹ 800 and ₹ 900 is given by $p(800 < X < 900)$.

When, $X = 800$,

$$\text{The corresponding, } z = \frac{X-\mu}{\sigma} \qquad \left(\because \begin{array}{l} X = 800 \\ \mu = 800 \\ \sigma = 50 \end{array}\right)$$

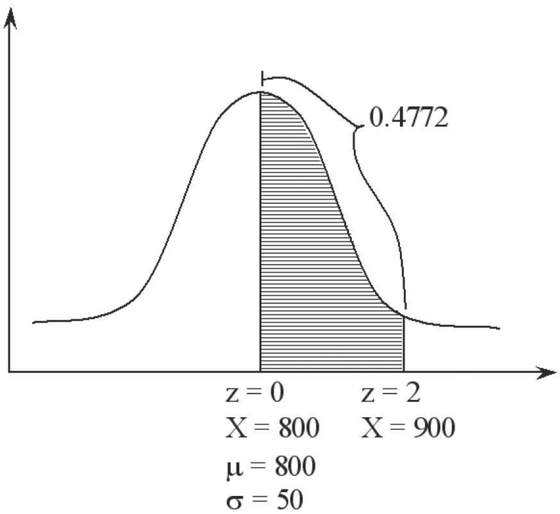
$$= \frac{800-800}{50} = 0$$

When, $X = 900$,

$$z = \frac{900-800}{50} = \frac{100}{50} = 2$$

$$\therefore \quad P(800 < X < 900) = P(0 < z < 2)$$

The area between $z = 0$ and $z = 2$ i.e., the shaded region in the graph given below.



Figure

$\therefore$ The required area and the area between $z = 0$ and $z = 2$ is,

$= 0.4772$ (From table, area $= 0.4772$ at $z = 2$)

$\therefore$ The number of workmen whose weekly wages are between ₹ 800 and ₹ 900.

$$= 0.4772 \times 1000$$

$$= 477.2$$

$$= 477 \text{ workmen.}$$

(ii) Less than ₹ 750

$$P(X < 750)$$

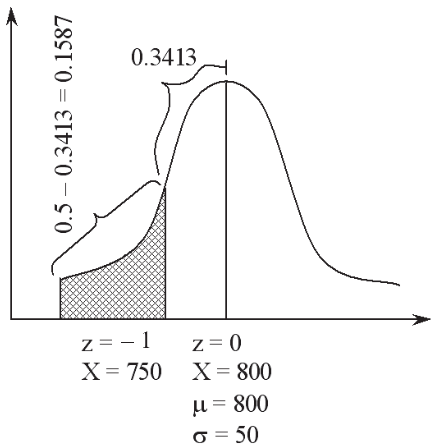
When,

$X = 750$, the corresponding $z = \frac{750-800}{50}$

$= \frac{-50}{50} = -1$

$\therefore \quad P(X < 750) = P(z < -1)$

The area left to the $z = -1$ i.e., the shaded region in the graph given below.



Figure

$\therefore$ The required area $= 0.5 - 0.3413$

$= 0.1587$

(At $z = -1$, area = 0.3413 from normal distribution table)

$\therefore$ The number of workmen whose weekly wages are less than ₹ 750

$= 0.1587 \times 1000 = 158.7$

$= 159$ workmen.

(iii) Between ₹ 700 and ₹ 750

$P(700 < X < 750)$

When, $X = 700$,

Corresponding z value, $z = \frac{700-800}{50}$

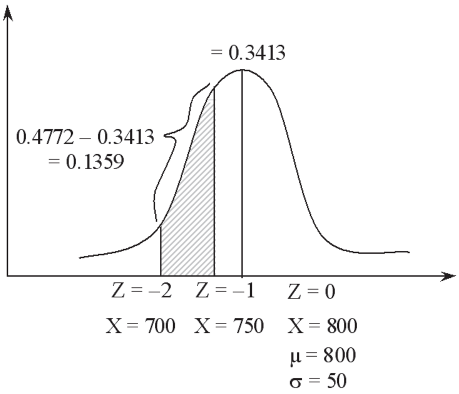
$= \frac{-100}{50} = -2$

When, $X = 750$,

$z = \frac{750-800}{50} = \frac{-50}{50} = -1$

$\therefore \quad P(700 < X < 750) = P(-2 < z < -1)$

The area between $z = -1$ and $z = -2$ i.e., the shaded region in the graph given below.



Figure

∴ The required area = 0.4772 – 0.3413 = 0.1359

(From normal distribution table, At $z = 1$, area = 0.3413 and $z = 2$, area = 0.4772)

∴ The number of workmen whose weekly wages are between ₹ 700 and ₹ 750

= 0.1359 × 1000 = 135.9 ≅ 136 workmen

(iv) More than ₹ 900

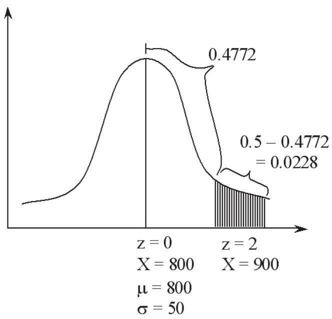
$P(X > 900)$

When, $X = 900$,

Corresponding $z = \dfrac{X-\mu}{\sigma} = \dfrac{900-800}{50} = \dfrac{100}{50} = 2$

∴ $P(X > 900) = P(z > 2)$

∴ The area right to the $z = 2$ i.e., the shaded region in the graph given below.



Figure

∴ The required area = 0.5 – 0.4772 = 0.0228. (At $z = 2$, area = 0.4772 from table)

∴ The number of workmen whose weekly wages are more than ₹ 900 = 0.0228 × 1000

= 22.8

≅ 23 workmen.

EXERCISE AND PRACTICE QUESTIONS

SHORT QUESTIONS

THEORY

Q1. Write the Mean of Binomial Distribution. [Refer, Q1]

Q2. Bring out the differences between Binomial and Poisson distribution. [Refer, Q2]

Q3. Properties of Normal Distribution. [Refer, Q5] May/June-18, Q8 (OU)

PROBLEMS

Q4. Comment on the following:

For a Binomial Distribution Mean = 8 and Variance = 12. [Refer Similar, Q4]

(Ans : q = 1.5).

Q5. 3 coins are tossed at the same time find the probability that 2 heads are occurred. [Refer Similar, Q6]

Q6. What is the probability of getting 4 heads when a coin is tossed 6 times? [Refer Similar, Q8]

Q7. A student obtained answers with mean μ = 2.4 and variance σ^2 = 3.2 for a certain problem given to him using binomial distribution comment on the result. [Refer Similar, Q4]

(Ans : 1.33 (inconsistent result)).

ESSAY QUESTIONS

THEORY

Q8. What is Binomial distribution? State its importance, applications and assumptions. [Refer, Q9]

Q9. Write about the conditions and constants under which binomial distribution is used. [Refer, Q11]

Q10. Briefly describe about fitting a binomial distribution along with an illustration. [Refer, Q12]

Q11. Define poisson distribution and state its importance. [Refer, Q17]

Q12. Write about the conditions and constants under which Poisson Distribution is used. [Refer, Q19]

Q13. What is Normal Distribution? Write the importance and applications of normal distribution. [Refer, Q24]

Q14. Explain in detail about central limit theorem. [Refer, Q26]

PROBLEMS

Q15. Four coins were tossed 150 times and the following results were obtained [Refer Similar, Q13]

x	0	1	2	3	4
f	12	50	54	30	4

Fit binomial distribution under the assumption that the coins are unbiased.

(Ans : Fitted Binomial Distribution:

x	0	1	2	3	4
f	9.375	37.5	56.25	37.5	9.375

).

Q16. In an accounting department of a bank 100 accounts are selected at random and examined for errors. The following result has been obtained,

Number of Errors	0	1	2	3	4	5	6
Number of Accounts	35	40	19	2	0	2	2

Does the data verify that the errors are distributed according to the poisson probability law? [Refer Similar, Q21]

(Ans : Fitted Poisson Distribution

Number of Errors	0	1	2	3	4	5	6
Number of Accounts	35	37	19	7	2	0	0

Errors are distributed according to the Poisson's probability law).

Q17. **The distribution of typing mistake committed by a typist is given below. Assuming a poisson model, find the expected frequencies.** [Refer Similar, Q21]

Mistakes per page:	0	1	2	3	4	5
Number of pages:	142	156	69	27	5	1

[Ans : Fitted Poisson distribution:

Number of Mistakes per Page	0	1	2	3	4	5
Number of Pages	147	147	74	25	6	1

]

Q18. **A random variable X is normally distributed with Mean (μ) = 12 and standard deviation (σ) = 2. Then find P(9.6 < X < 13.8) [given that $\frac{x}{\sigma}$ = 0.9, A = 0.3159 and for $\frac{x}{\sigma}$ = 1.2, A = 0.3849].** [Refer Similar, Q32]

[Ans : 0.7008].

Q19. **A workshop produces 2000 units of an item per day. The average weight of units is 130kg with a standard deviation of 10kg. Assuming normal distribution, how many units are expected to weight less than 142kg?** [Refer Similar, Q32]

[Ans : 1,770 approx].

Q20. **If X is normally distributed with mean 70 and standard deviation 16. Find,**

(i) P(38 ≤ X ≤ 46)

(ii) P(82 ≤ X ≤ 94). [Refer Similar, Q31]

[Ans : P(38 ≤ x ≤ 46) = 0.044; P(82 ≤ X ≤ 94) = 0.1598].

INTERNAL ASSESSMENT/EXAM

I Multiple Choice

1. __________ is a discrete probability distribution developed by a Swiss mathematician. [ ]
   (a) Binomial distribution (b) Poisson distribution
   (c) Normal distribution (d) None of the above
2. Mean of the Poisson Distribution is given by __________. [ ]
   (a) np (b) p
   (c) λ (d) pq
3. __________ is also known as Bernoulli distribution. [ ]
   (a) Gamma distribution (b) Beta distribution
   (c) Normal distribution (d) Binomial distribution
4. _______ plays a major role in statistical theory and it has a widest application area than other distribution mechanism. [ ]
   (a) Rectangular distribution (b) Gamma distribution
   (c) Normal distribution (d) Beta distribution
5. The mean of binomial distribution is denoted by __________. [ ]
   (a) σ^2 (b) μ
   (c) np (d) pq
6. Poisson distribution satisfies two essential properties such as __________. [ ]
   (a) $f(x) \geq o$ (b) $\Sigma f(x) = 1$
   (c) Both (a) and (b) (d) None of the above
7. The variance of binomial distribution is __________ [ ]
   (a) $\lambda^2 = \sigma^2$ (b) $\sigma = \sqrt{\lambda}$
   (c) $\sigma^2 = npq$ (d) None of the above
8. The two parameters of normal distribution are ____________. [ ]
   (a) Mean and standard deviation (b) Standard deviation and variance
   (c) Mean and variance (d) None of the above
9. Poission distribution is used generally to approximate the __________ when 'n' value is large and 'p' value is very small. [ ]
   (a) Binomial distribution (b) Gamma distribution
   (c) Normal distribution (d) None of the above
10. In a __________ mean, median and mode are all equal or they coincide with each other. [ ]
   (a) Binomial distribution (b) Beta distribution
   (c) Normal distribution (d) Gamma distribution

II Fill in the Blanks

1. Mean of Binomial distribution is __________.
2. Probability mass function of poisson distribution is __________.
3. Variance of poisson distribution is __________.
4. The probability mass function of Binomial distribution is ___________.
5. The binomial distribution except that it can perform any number of trails for fixed number of success is called as __________.
6. Mean in poisson distribution is calculated by using __________ formula.
7. The sum of infinite probabilities in poisson distribution is denoted as __________.
8. Normal distribution is also known as __________.
9. The total area under the normal curve for normal probability distribution is __________.
10. Binomial distribution produces successes and failures where as poission distribution produces successes which referred as __________.

KEY

I. Multiple Choice

1. (a)
2. (c)
3. (d)
4. (c)
5. (b)
6. (c)
7. (c)
8. (c)
9. (a)
10. (c)

II. Fill in the Blanks

1. np
2. $P(x,\lambda) = \dfrac{e^{-\lambda}\lambda^{x}}{x!}$
3. λ
4. $P(x) = {}^{n}C_{x}p^{x}q^{n-x}$
5. Negative binomial distribution
6. $\lambda = nxp$
7. P(0) + P(1) + P(2) + ...+ P(∞) = 1
8. Gaussian distribution
9. 1
10. Occurances.

III Very Short Questions and Answers

Q1. What is Binomial Distribution?

Answer :

Binomial distribution is a discrete probability distribution developed by a Swiss mathematician, 'James Bernoulli'. Binomial distribution is applicable in case of repeated trials such as,

(i) Number of applications received for a junior assistant post during a particular period of time.

(ii) Number of births taking place in a hospital.

Q2. Define Poisson Distribution.

Answer :

A random variable 'x' is said to have a poisson distribution if it assumes only positive values and its Probability Mass Function (P.M.F) is given by, p(x) = $\frac{e^{-\lambda}\lambda^{x}}{x!}$.

Q3. Write a short note on the conditions under which poisson distribution is used.

Answer :

Poisson distribution is a limiting case when,

(i) $n \to \infty$ i.e., number of trails is very large.

(ii) $P \to 0$ i.e., Probability of success for each trial is very small.

(iii) $np = \lambda$ is a finite constant.

Q4. State Bernoullis Distribution.

Answer :

Bernoulli's distribution is an experiment that can have either of the two possible outcomes (i.e., success or failure).

Q5. What do you understand by normal distribution?

Answer :

In normal distribution, the probability of occurrence of values of random variables are calculated within a range or interval whereas the probability of a particular value cannot be calculated and is always assumed to be zero. The probability distribution used mostly for dealing with quantities whose magnitude is continuously variable is normal distribution. Hence, it is called as 'Normal probability distribution'. It plays a prominent role in statistical theory and practice.